Algebra II\ Trig Ch. 14: 14.3-14.6 Quiz

Use definitions to prove each identity.

1. $\sin^2\theta+\cos^2\theta=\sec^2\theta-\tan^2\theta$

$$1=\frac{1}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}$$

$$1=\frac{1-\sin^2\theta}{\cos^2\theta}$$

$$1=\frac{\cos^2\theta}{\cos^2\theta}$$

$$1=1$$

2. $1+\cot^2\theta=\csc^2\theta$

$$1+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\frac{1}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

Write each expression in terms of a single trigonometric function.

3. $\dfrac{\tan\theta}{\sin\theta}$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{1}}$$

$$\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}$$

$$\frac{1}{\cos\theta}=\sec\theta$$

4. $\csc\theta\cos\theta\sin\theta$

$$\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{1}\cdot\frac{\sin\theta}{1}$$

$$\cos\theta$$

Write each expression in term of only one trigonometric function. Then simplify, if possible.

5. $\sec^2\theta - \tan^2\theta$

$$\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$\frac{1-\sin^2\theta}{\cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta}$$

$$1$$

6. $\sec\theta(\sec\theta - \cos\theta)$

$$\frac{1}{\cos\theta}\left(\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\right)$$

$$\frac{1}{\cos^2\theta} - \frac{\cos\theta}{\cos\theta}$$

$$\frac{1}{\cos^2\theta} - 1$$

$$\sec^2\theta - 1$$

$$\tan^2\theta$$

Find the exact value of each expression.

7. $\sin(45° + 60°)$

$\sin 45° \cos 60° + \cos 45° \sin 60°$

$\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{\sqrt{2} + \sqrt{6}}{4}$

8. $\cos 195°$

$45° + 135°$

$\cos(135° + 60°)$

$\cos 135° \cos 60° - \sin 135° \sin 60°$

$-\frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$\frac{-\sqrt{2} - \sqrt{6}}{4}$

Find the rotation matrix for ~~120°~~ 150°

9.

$$\begin{bmatrix} \cos 150^\circ & -\sin 150^\circ \\ \sin 150^\circ & \cos 150^\circ \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

Simplify

10. $\dfrac{\cos 2\theta}{\cos\theta - \sin\theta} - \cos\theta$

$$\frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} - \cos\theta$$

$$\frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta - \sin\theta} - \cos\theta$$

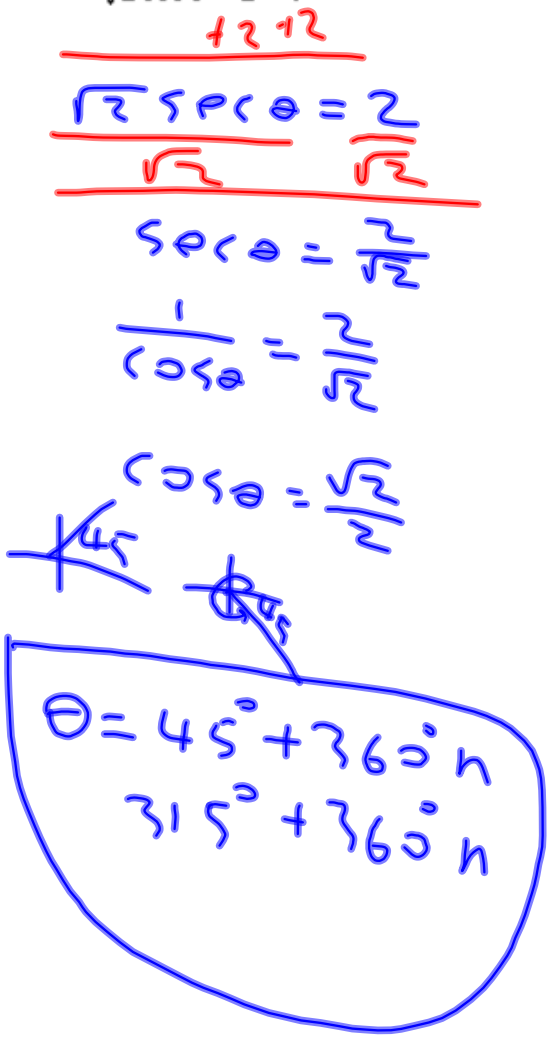
$$\cos\theta + \sin\theta - \cos\theta = \sin\theta$$

Write each expression in terms of trigonometric functions of θ rather than multiples of θ.

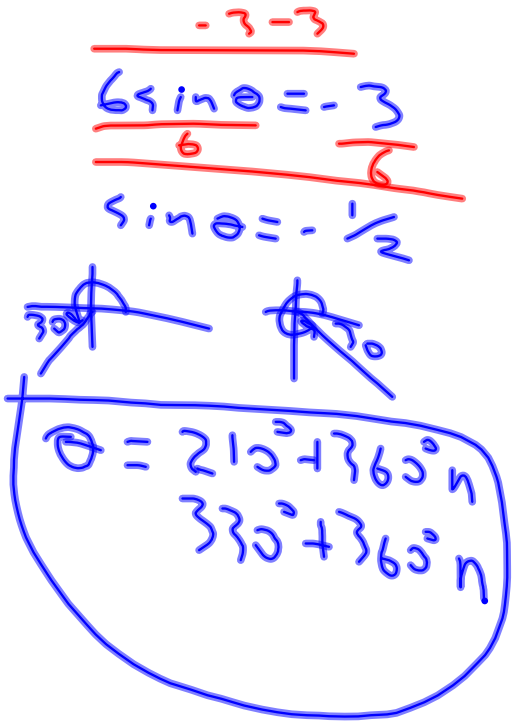
11. $\dfrac{\cos^2\theta}{\sin 2\theta} = \dfrac{\cos^2\theta}{2\sin\theta\cos\theta} = \dfrac{\cos\theta}{2\sin\theta} = \dfrac{\cot\theta}{2}$

Find all solutions of each equation

12. $\sqrt{2}\sec\theta - 2 = 0$

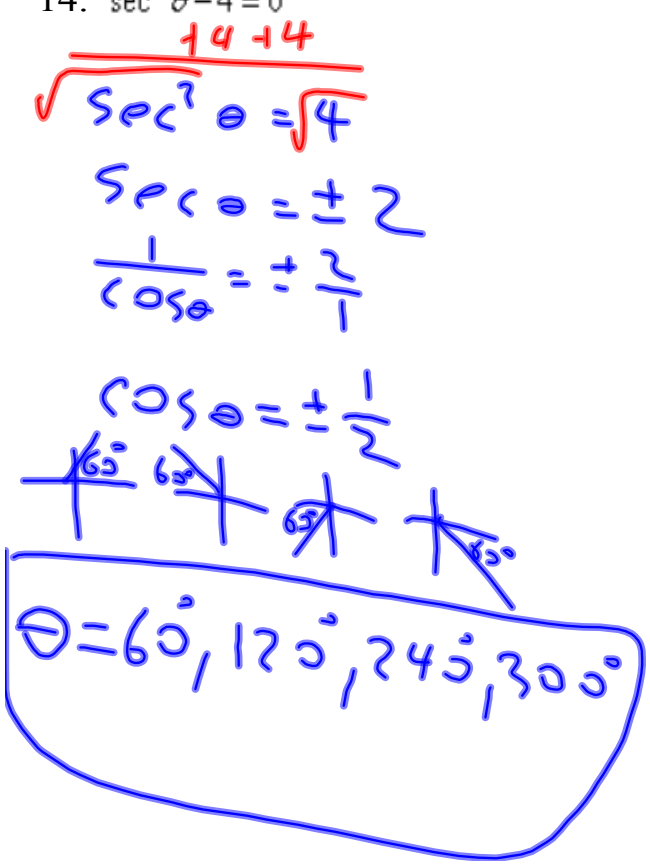


13. $6\sin\theta + 3 = 0$

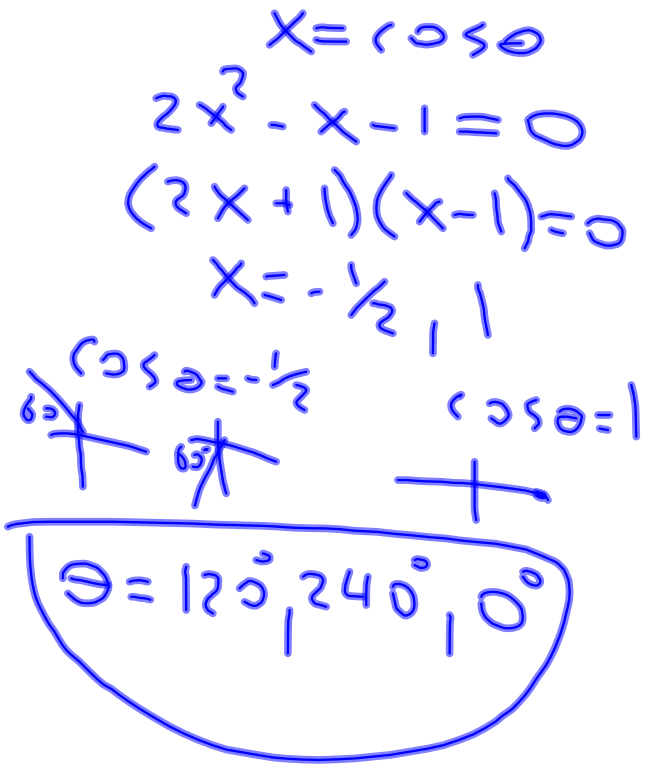


Find the exact solutions of each equation for $0° \le \theta < 360°$

14. $\sec^2\theta - 4 = 0$



15. $2\cos^2\theta - \cos\theta - 1 = 0$



16. $2\sin\theta\cos\theta = \tan\theta$

$2\sin\theta\cos\theta = \frac{\sin\theta}{\cos\theta}$

$2\sin\theta\cos^2\theta = \sin\theta$

$-\sin\theta \quad -\sin\theta$

$2\sin\theta\cos^2\theta - \sin\theta = 0$

$\sin\theta(2\cos^2\theta - 1) = 0$

$\sin\theta = 0 \qquad 2\cos^2\theta - 1 = 0$

$+1 \quad +1$

$2\cos^2\theta = 1$

$\frac{2}{2} \quad \frac{1}{2}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{1}{2}}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = 0°, 180°, 45°, 135°, 225°, 315°$

17. $2\sin^2\theta = 1 - \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$x = \sin\theta$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$x = \frac{1}{2}, -1$

$\sin\theta = \frac{1}{2} \qquad \sin\theta = -1$

$\theta = 30°, 150°, 270°$

Use the information given to find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\sin\frac{\theta}{2}$, and $\cos\frac{\theta}{2}$.

18. $90^\circ \le \theta \le 180^\circ; \sin\theta = \frac{5}{13}$

$\cos\theta = -\frac{12}{13}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2}{1}\cdot\frac{5}{13}\cdot\frac{-12}{13}$

$\sin 2\theta = -\frac{120}{169}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169}$

$\cos 2\theta = \frac{119}{169}$

$90^\circ \le \theta \le 180^\circ$

$45^\circ \le \frac{\theta}{2} \le 90^\circ$

$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\left(-\frac{12}{13}\right)}{2}}$

$= \sqrt{\frac{\frac{25}{13}}{2}}$

$= \sqrt{\frac{25}{13}\cdot\frac{1}{2}}$

$\sin\frac{\theta}{2} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}}$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$= \sqrt{\frac{1+\left(-\frac{12}{13}\right)}{2}}$

$= \sqrt{\frac{\frac{1}{13}}{2}}$

$= \sqrt{\frac{1}{13}\cdot\frac{1}{2}}$

$\cos\frac{\theta}{2} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}}$