14.3- Fundamental Trigonometric Identities

Fundamental Identities

Ratio Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Prove the identity $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\dfrac{\frac{1}{1}}{\frac{\sin\theta}{\cos\theta}} =$

$\dfrac{1}{1} \cdot \dfrac{\cos\theta}{\sin\theta} =$

$\dfrac{\cos\theta}{\sin\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\sin\theta = \tan\theta\cos\theta$

$\sin\theta = \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{1}$

$\sin\theta = \sin\theta$

Prove the identity $\tan^2\theta + 1 = \sec^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{1}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

Write $\dfrac{\cos^2\theta}{1-\sin\theta}$ in terms of a single trig function.

$$\frac{1-\sin^2\theta}{1-\sin\theta} = \frac{(1+\sin\theta)(1-\sin\theta)}{1-\sin\theta}$$

$$1+\sin\theta$$

Write $\frac{1}{\sec^2\theta}$ in terms of $\sin\theta$

$$\frac{\frac{1}{1}}{\frac{1}{\cos^2\theta}} = \frac{1}{1}\cdot\frac{\cos^2\theta}{1} = \cos^2\theta = \boxed{1-\sin^2\theta}$$

Homework

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14) $\tan\theta \cos\theta$

$$\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}$$

$$\sin\theta$$

16) $\tan\theta \sec\theta \sin\theta$

$$\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\tan^2\theta$$

18) $\sec\theta \cos^2\theta$

$\frac{1}{\cancel{\cos\theta}} \cdot \frac{\cos^{\cancel{2}}\theta}{1}$

$\cos\theta$

20) $\left(\frac{\cos^3\theta}{\sin\theta}\right)\sec^2\theta$

$$\frac{\cos^3\theta}{\sin\theta}\cdot\frac{1}{\cos^2\theta}=\frac{\cos\theta}{\sin\theta}$$

$$\cot\theta$$

22) $\dfrac{\cos\theta}{\cot\theta}$

$$\frac{\frac{\cos\theta}{1}}{\frac{\cos\theta}{\sin\theta}} \rightarrow \frac{\cos\theta}{1} \cdot \frac{\sin\theta}{\cos\theta}$$

$\sin\theta$

24) $\dfrac{\sec^2\theta}{\tan^2\theta}$

$$\frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} \rightarrow \frac{1}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\csc^2\theta$$

26) $\left(\frac{\cos\theta}{\tan\theta}\right)\sin\theta$

$$\frac{\frac{\cos\theta}{1}}{\frac{\sin\theta}{\cos\theta}} \cdot \sin\theta$$

$$\frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{1} = \cos^2\theta$$

28) $(1-\sin^2\theta)(1+\sec^2\theta)$

$\cos^2\theta\left(1+\frac{1}{\cos^2\theta}\right)$

$\cos^2\theta+\frac{\cos^2\theta}{\cos^2\theta}$

$\cos^2\theta+1$

30) $\frac{\tan\theta}{\sin\theta}$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{1}}$$

$$\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{1}{\cos\theta}$$

32) $\cot^2\theta$

$$\frac{\cos^2\theta}{\sin^2\theta} = \frac{1-\sin^2\theta}{\sin^2\theta}$$

34) $\dfrac{1}{\sec^2\theta}$

$$\frac{\frac{1}{1}}{\frac{1}{\cos^2\theta}} \rightarrow \frac{1}{1} \cdot \frac{\cos^2\theta}{1} = \cos^2\theta$$

$1 - \sin^2\theta$

36) $\dfrac{\csc\theta}{\sec\theta} = \cot\theta$

$$\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$$

$$\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{1} = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

38) $\dfrac{\cot^2\theta}{\csc^2\theta} = \cos^2\theta$

$$\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}} = \cos^2\theta$$

$$\frac{\cos^2\theta}{\cancel{\sin^2\theta}} \cdot \frac{\cancel{\sin^2\theta}}{1} = \cos^2\theta$$

$$\cos^2\theta = \cos^2\theta$$

40) $\tan^2\theta = \sin^2\theta \sec^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{1} \cdot \frac{1}{\cos^2\theta}$$

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}$$

42) $\dfrac{\csc\theta}{\sin\theta} = \csc^2\theta$

$$\frac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{1}} = \frac{1}{\sin^2\theta}$$

$$\frac{1}{\sin\theta}\cdot\frac{1}{\sin\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{1}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

44) $\dfrac{\sin\theta}{1-\cos^2\theta} = \csc\theta$

$\dfrac{\sin^1\theta}{\sin^2\theta} = \dfrac{1}{\sin\theta}$

$\dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta}$

46) $\csc\theta(1-\cos^2\theta) = \sin\theta$

$$\frac{1}{\sin\theta} \cdot \frac{\sin^2\theta}{1} = \sin\theta$$

$$\sin\theta = \sin\theta$$

48) $(\cot\theta)(\csc\theta)(\sec\theta) = \csc^2\theta$

$$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{1}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

50) $\tan^2\theta - 2\sec\theta\sin\theta$

$$\tan^2\theta - \frac{2}{1}\cdot\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{1}$$

$$\tan^2\theta - \frac{2\sin\theta}{\cos\theta}$$

$$\tan^2\theta - 2\tan\theta$$

52) $\csc\theta$

$$\frac{1}{\sin\theta} = \frac{1}{\pm\sqrt{1-\cos^2\theta}}$$

$$\sqrt{\sin^2\theta} = \sqrt{1-\cos^2\theta}$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

54) $\cot\theta$

$$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\pm\sqrt{1-\cos^2\theta}}$$