

Chapter 14 Review

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Und.

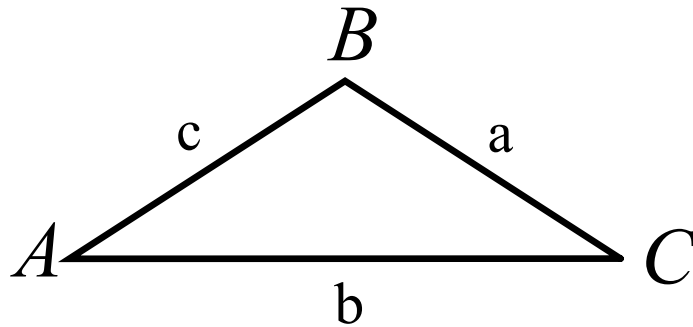
Area of a Triangle

The area, K , of $\triangle ABC$ is given by the equation below.

$$K = \frac{1}{2}bc \sin A$$

$$K = \frac{1}{2}ac \sin B$$

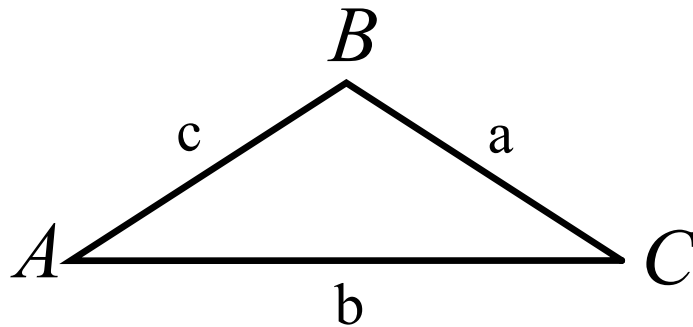
$$K = \frac{1}{2}ab \sin C$$



Law of Sines

For $\triangle ABC$, the law of sines states the following:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



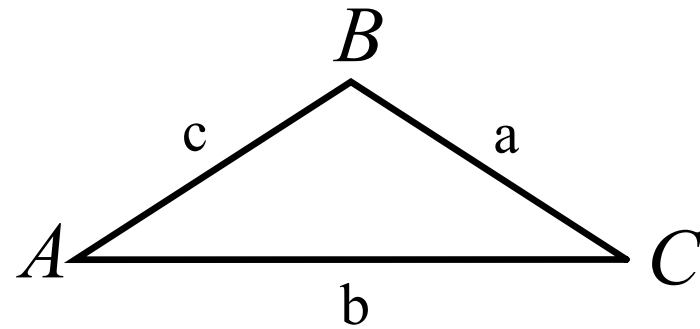
Law of Cosines

In any triangle $\triangle ABC$, the law of cosines states the following:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Fundamental Identities

Ratio Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Pythagorean Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Sum and Difference Identities

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

Rotation Matrix

If $P(x, y)$ is any point in a plane, then the coordinates of the image of point $P'(x', y')$ after a rotation of θ degrees about the origin can be found by using a *rotation matrix* as follows:

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Double-Angle Identities

$$\sin 2\theta = 2 \sin\theta \cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$

Half-Angle Identities

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos\theta}{2}}$$

Choose + or - depending on the sign of the value for

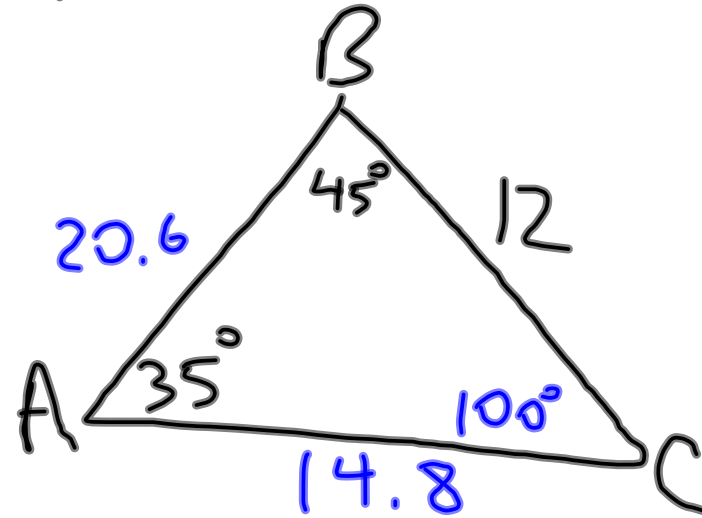
$$\sin \frac{\theta}{2} \text{ or } \cos \frac{\theta}{2}.$$

Homework

Pg. 930-932 #1-43 odd

$$1) A = 35^\circ, B = 45^\circ, a = 12$$

$$\begin{aligned} C &= 100^\circ \\ b &= 14.8 \\ c &= 20.6 \end{aligned}$$

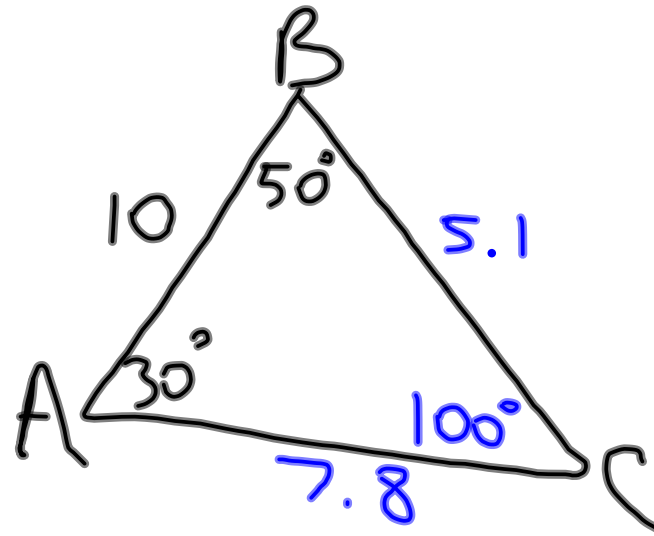


$$\frac{b}{\sin 45^\circ} = \frac{12}{\sin 35^\circ}$$

$$\frac{c}{\sin 100^\circ} = \frac{12}{\sin 35^\circ}$$

$$3) A = 30^\circ, c = 10, B = 50^\circ$$

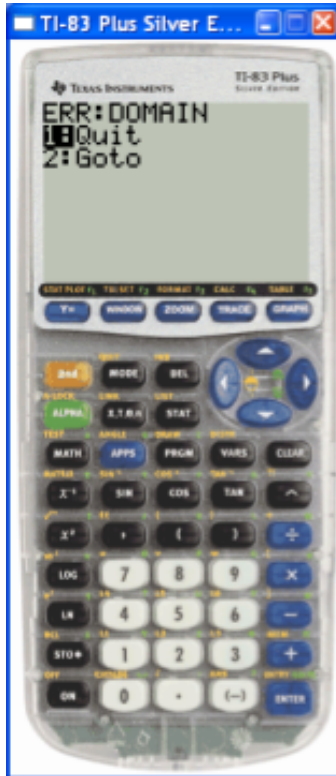
$$\begin{aligned} C &= 100^\circ \\ a &= 5.1 \\ b &= 7.8 \end{aligned}$$



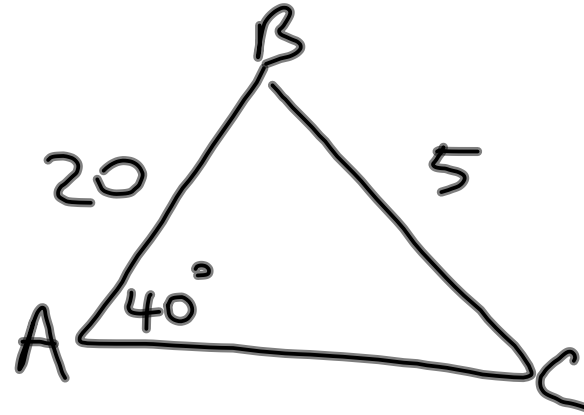
$$\frac{a}{\sin 30^\circ} = \frac{10}{\sin 100^\circ}$$

$$\frac{b}{\sin 50^\circ} = \frac{10}{\sin 100^\circ}$$

$$5) A = 40^\circ, c = 20, a = 5$$



$$\frac{\sin C}{20} = \frac{\sin 40^\circ}{5}$$



$$C = \sin^{-1}\left(\frac{20 \sin 40^\circ}{5}\right)$$

0 Triangles

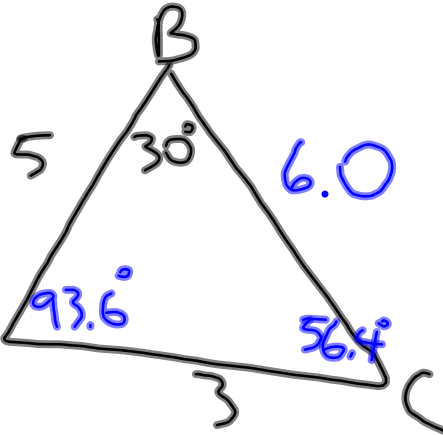
$$7) B=30^\circ, b=3, c=5$$

$$\begin{aligned} A &= 93.6^\circ \\ C &= 56.4^\circ \\ a &= 6.0 \end{aligned}$$

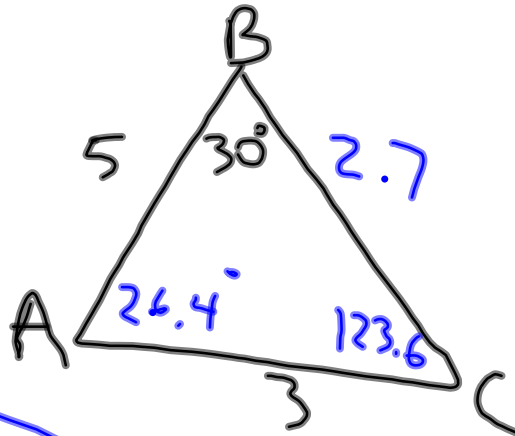
$$\frac{\sin C}{5} = \frac{\sin 30^\circ}{3}$$

$$C = \sin^{-1}\left(\frac{5 \sin 30^\circ}{3}\right)$$

$$\frac{a}{\sin 93.6^\circ} = \frac{3}{\sin 30^\circ}$$



$$\frac{a}{\sin 26.4^\circ} = \frac{3}{\sin 30^\circ}$$



$$\begin{aligned} A &= 26.4^\circ \\ C &= 123.6^\circ \\ a &= 2.7 \end{aligned}$$

2 Triangles

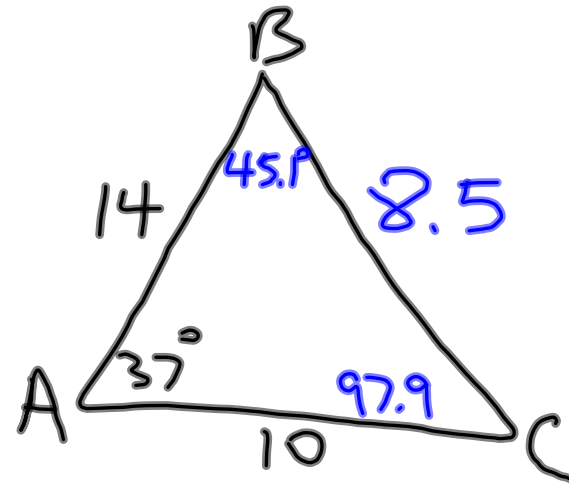
9) $A = 37^\circ$, $b = 10$, $c = 14$

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$a = 8.5$

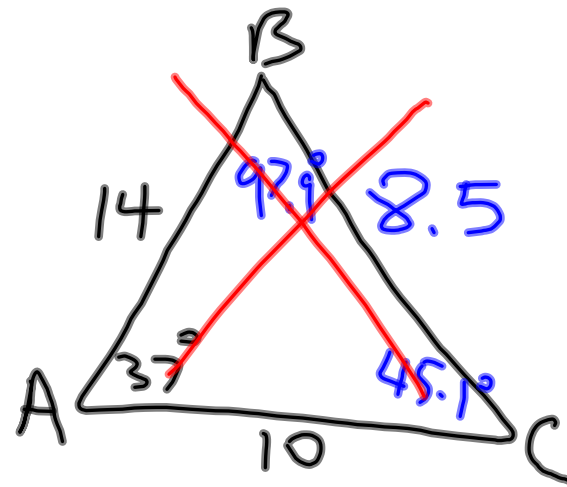
$B = 45.1^\circ$

$C = 97.9^\circ$



$$a = \sqrt{10^2 + 14^2 - 2(10)(14)\cos 37^\circ}$$

$$\frac{\sin B}{10} = \frac{\sin 37^\circ}{8.5}$$

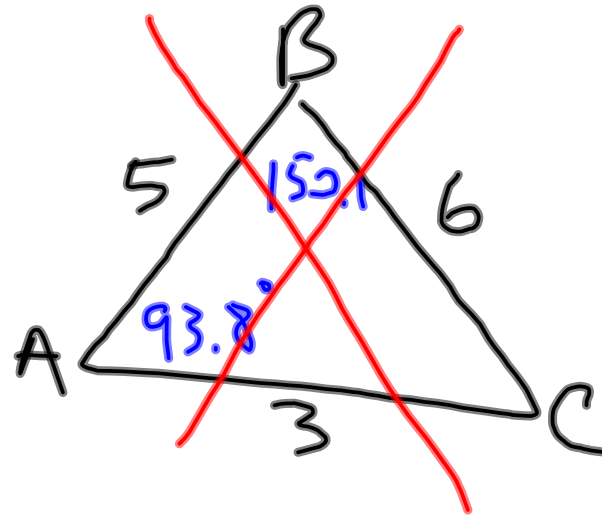
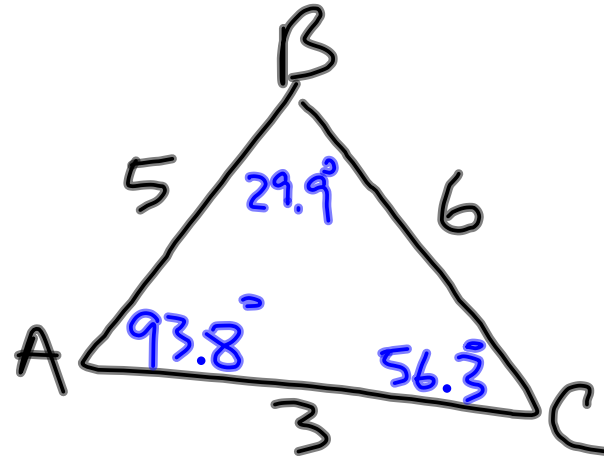


$$11) a=6, b=3, c=5$$

$$\begin{aligned} A &= 93.8^\circ \\ B &= 29.9^\circ \\ C &= 56.3^\circ \end{aligned}$$

$$A = \cos^{-1} \left(\frac{6^2 - 5^2 - 3^2}{-2(5)(3)} \right)$$

$$\frac{\sin B}{3} = \frac{\sin 93.8^\circ}{6}$$



$$13) A = 35^\circ, C = 30, a = 20$$

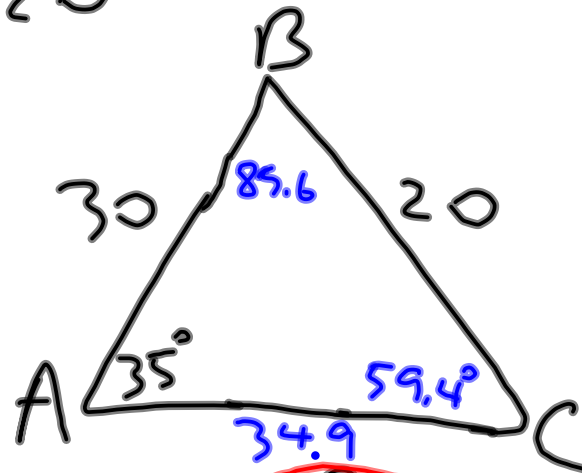
$$B = 89.6^\circ$$

$$C = 59.4^\circ$$

$$b = 34.9$$

$$\frac{\sin C}{30} = \frac{\sin 35^\circ}{20}$$

$$\frac{b}{\sin 89.6^\circ} = \frac{20}{\sin 35^\circ}$$

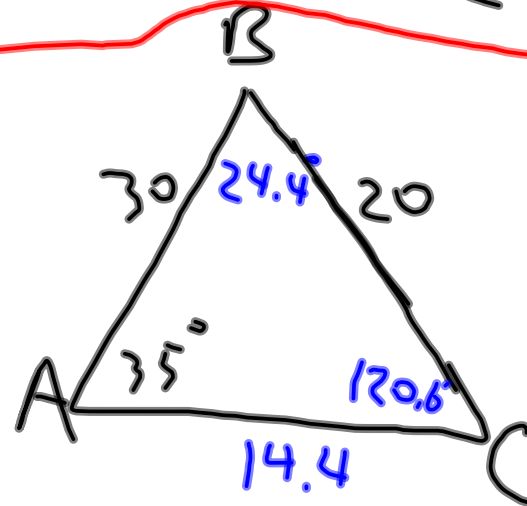


$$B = 24.4^\circ$$

$$C = 120.6^\circ$$

$$b = 14.4$$

$$\frac{b}{\sin 24.4^\circ} = \frac{20}{\sin 35^\circ}$$



$$15) \csc \theta \tan \theta$$

$$\frac{1}{\cancel{\sin \theta}} \cdot \frac{\cancel{\sin \theta}}{\cos \theta}$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$17) \frac{-3 \tan^2 \theta}{1 + \sec^2 \theta}$$

$$\frac{-3 \tan^2 \theta}{1 + \tan^2 \theta + 1}$$

$$\frac{-3 \tan^2 \theta}{2 + \tan^2 \theta}$$

$$19) \sin(60^\circ + 270^\circ)$$

$$\sin 60^\circ \cos 270^\circ + \cos 60^\circ \sin 270^\circ$$

$$\frac{\sqrt{3}}{2} \cdot 0 + \frac{1}{2} \cdot (-1)$$

$$\left(-\frac{1}{2}\right)$$

$$21) \cos(120^\circ - 135^\circ)$$

$$\cos 120^\circ \cos 135^\circ + \sin 120^\circ \sin 135^\circ$$
$$\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$
$$\frac{\sqrt{2} + \sqrt{6}}{4}$$

$$23) \cos 75^\circ$$
$$\cos(30^\circ + 45^\circ)$$

$$\cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

$$25) \sin(-15^\circ)$$

$$\sin(30^\circ - 45^\circ)$$

$$\sin 30^\circ \cos 45^\circ - (\cos 30^\circ \sin 45^\circ)$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

27) 120° rotation $(-2, 7)$



$$\begin{bmatrix} \cos 120^\circ & -\sin 120^\circ \\ \sin 120^\circ & \cos 120^\circ \end{bmatrix}$$

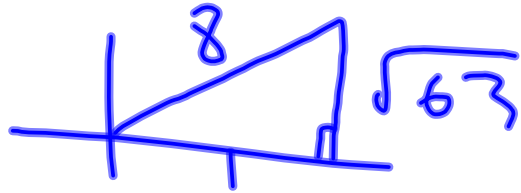
$$\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -5.06 \\ -5.23 \end{bmatrix}$$

$$(-5.06, -5.23)$$

Given $0^\circ \leq \theta \leq 90^\circ$; $\cos \theta = \frac{1}{8}$



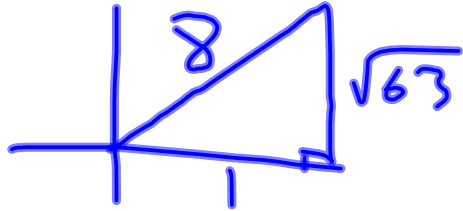
$$\sin \theta = \frac{\sqrt{63}}{8}$$

29) $\sin 2\theta$

$$2 \sin \theta \cos \theta$$

$$\frac{2}{1} \cdot \frac{\sqrt{63}}{8} \cdot \frac{1}{8} = \frac{\sqrt{63}}{32}$$

Given $0^\circ \leq \theta \leq 90^\circ$; $\cos \theta = \frac{1}{8}$



$$\sin \theta = \frac{\sqrt{63}}{8}$$

31) $\sin \frac{\theta}{2}$

$$\frac{1}{8} \leq \frac{1}{8} \leq \frac{1}{2}$$

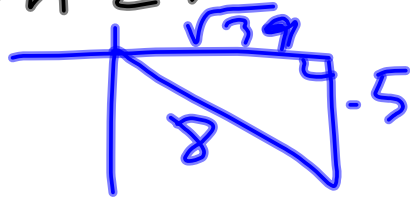
$$0 \leq \frac{\theta}{2} \leq 45^\circ$$

$$= \sqrt{\frac{1 - \frac{1}{8}}{2}}$$

$$= \sqrt{\frac{1 - \frac{1}{8}}{2}}$$

$$= \sqrt{\frac{1 - \frac{1}{8}}{2}} = \sqrt{\frac{\frac{7}{8}}{2}} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

Given $270^\circ \leq \theta \leq 360^\circ$; $\sin \theta = -\frac{5}{8}$



$$\cos \theta = \frac{\sqrt{39}}{8}$$

33) $\cos 2\theta$

$$1 - 2 \sin^2 \theta$$

$$1 - \left(2 \cdot \left(-\frac{5}{8} \right)^2 \right)$$

$$1 - \frac{2 \cdot 25}{64}$$

$$\frac{32}{32} - \frac{25}{32} = \frac{7}{32}$$

$$\cos^2 \theta - \sin^2 \theta$$

$$\left(\frac{\sqrt{39}}{8} \right)^2 - \left(-\frac{5}{8} \right)^2$$

$$\frac{39}{64} - \frac{25}{64}$$

$$\frac{14}{64} = \frac{7}{32}$$

$$35) \quad 270^\circ \leq \theta \leq 360^\circ$$

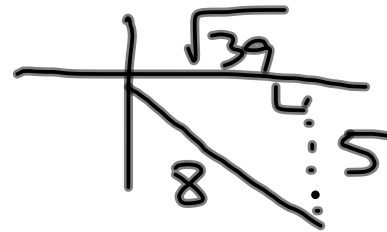
$$135^\circ \leq \frac{\theta}{2} \leq 180^\circ$$

$$\cos \frac{\theta}{2} = -\sqrt{\frac{1 + \frac{\sqrt{39}}{8}}{2}}$$

Quad. II

$$\cos \frac{\theta}{2} = -\sqrt{\frac{8 + \sqrt{39}}{16}}$$
$$= -\frac{\sqrt{8 + \sqrt{39}}}{4}$$

$$\sin \theta = -\frac{5}{8}$$



$$\cos \theta = \frac{\sqrt{39}}{8}$$

$$37) \cos 4\theta$$

$$\cos 2(2\theta)$$

$$\cos^2 2\theta - \sin^2 2\theta$$

$$(\cos^2 \theta - \sin^2 \theta)^2 - (2\sin \theta \cos \theta)^2$$

$$(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) - 4\sin^2 \theta \cos^2 \theta$$

$$\cos^4 \theta - 2\cos^2 \theta \sin^2 \theta + \sin^4 \theta - 4\sin^2 \theta \cos^2 \theta$$

$$\cos^4 \theta - 6\cos^2 \theta \sin^2 \theta + \sin^4 \theta$$

$$39) \quad \sin^2 \theta + \sin \theta = 2$$

$$\sin^2 \theta + \sin \theta - 2 = 0$$

$$x = \sin \theta$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1$$

$$\sin \theta = -2$$

$$\sin \theta = 1$$

$$\theta = 90^\circ$$

†

$$41) \cos^2 \theta - \sin^2 \theta + 1 = 0$$

$$\cos^2 \theta - (1 - \cos^2 \theta) + 1 = 0$$

$$\cos^2 \theta - 1 + \cos^2 \theta + 1 = 0$$

$$\frac{2\cos^2 \theta}{2} = \frac{0}{2}$$

$$\sqrt{\cos^2 \theta} = \sqrt{0}$$

$$\cos \theta = 0$$

$$\theta = 90^\circ, 270^\circ$$

$$43) 4\sin^2\theta - 2(\sqrt{2}+1)\sin\theta + \sqrt{2} = 0$$

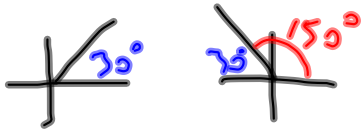
$$X = \sin\theta$$

$$4x^2 - 2(\sqrt{2}+1)x + \sqrt{2} = 0$$

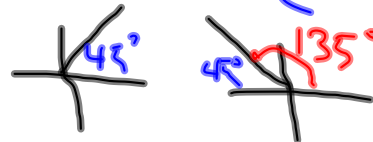
$$(2x - 1)(2x - \sqrt{2}) = 0$$

$$X = \frac{1}{2}, \frac{\sqrt{2}}{2}$$

$$\sin\theta = \frac{1}{2}$$



$$\sin\theta = \frac{\sqrt{2}}{2}$$



$$\theta = 30^\circ, 150^\circ, 45^\circ, 135^\circ$$