

2.5- Inverses of Functions

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x) .

The domain of the inverse is the range of the original relation.

The range of the inverse is the domain of the original relation.

Find the inverse of each relation. State whether the relation is a function. State whether the inverse is a function.

Relation
 $(3, 5), (6, 10), (9, 15)$

yes

Inverse
 $(5, 3), (10, 6), (15, 9)$

yes

Find an equation for the inverse of $y = 4x - 5$

$$\begin{aligned} X &= 4y - 5 \\ +5 & \qquad +5 \\ \hline X+5 &= 4y \\ \frac{X+5}{4} &= \frac{4y}{4} \\ \hline y^{-1} &= \frac{X+5}{4} \end{aligned}$$

Find the inverse of $f(x) = 5x + 1$

$$y = 5x + 1$$

$$x = 5y + 1$$

$$\frac{x-1}{5} = \frac{5y}{5}$$

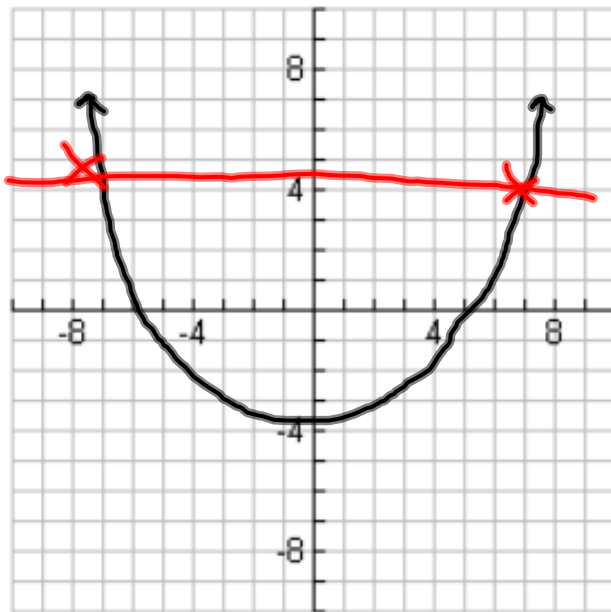
$$y = \frac{x-1}{5}$$

$$f^{-1}(x) = \frac{x-1}{5}$$

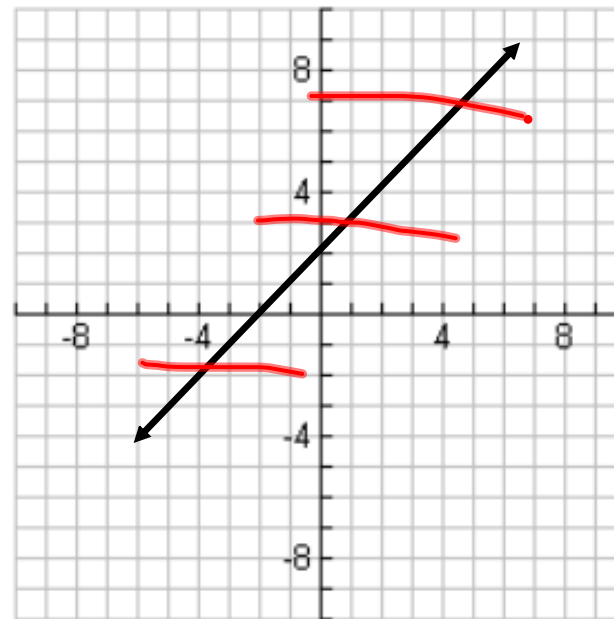
Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function at no more than one point.

No



Yes



Composition and Inverses

If f and g are functions and $(f \circ g)(x) = (g \circ f)(x) = I(x) = x$
then f and g are inverses of one another.

$$\text{If } f(g(x)) = x \quad \text{And } g(f(x)) = x$$

Then f and g are inverses of
each other

$$f(x) = -5x + 7 \quad g(x) = -\frac{1}{5}x + \frac{7}{5}$$

$$\begin{aligned} f \circ g(x) &= f(g(x)) = -5\left(-\frac{1}{5}x + \frac{7}{5}\right) + 7 \\ &= x - 7 + 7 \\ &= x \end{aligned}$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) = -\frac{1}{5}(-5x + 7) + \frac{7}{5} \\ &= x - \frac{7}{5} + \frac{7}{5} \\ &= x \end{aligned}$$

Homework

Pg. 122-123 #14-18 even, 24, 26-28 all,
38-40 all, 52

$$38) h(x) = \frac{1}{4}(x-1)$$

$$h(x) = \frac{1}{4}x - \frac{1}{4}$$

$$y = \frac{1}{4}x - \frac{1}{4}$$

$$x = \frac{1}{4}y - \frac{1}{4}$$

$$4x + 1 = y$$

$$4x + 1 = y$$

$$h^{-1}(x) = 4x + 1$$

$$\begin{aligned} h \circ h^{-1}(x) &= \frac{1}{4}(4x+1) - \frac{1}{4} \\ &= x + \frac{1}{4} - \frac{1}{4} \\ &= x \end{aligned}$$

$$\begin{aligned} h^{-1} \circ h(x) &= 4\left(\frac{1}{4}x - \frac{1}{4}\right) + 1 \\ &= x - 1 + 1 \\ &= x \end{aligned}$$

$$39) g(x) = \frac{1}{2}(x+2) - 3$$

$$g(x) = \frac{1}{2}x + 1 - 3$$

$$g(x) = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x - 2$$

$$x = \frac{1}{2}y - 2$$

$$\begin{array}{l} +2 \qquad \qquad +2 \\ \hline 2 \left[x+2 = \frac{1}{2}y \right] \end{array}$$

$$2x+4 = y$$

$$g^{-1}(y) = 2x+4$$

$$g \circ g^{-1}(x) = \frac{1}{2}(2x+4) - 2$$

$$= x + 2 - 2$$

$$= x$$

$$g^{-1} \circ g(x) = 2\left(\frac{1}{2}x - 2\right) + 4$$

$$= x - 4 + 4$$

$$= x$$

$$40) h(x) = \frac{3}{2}(x-3) + 2$$

$$h(x) = \frac{3}{2}x - \frac{9}{2} + \frac{4}{2}$$

$$h(x) = \frac{3}{2}x - \frac{5}{2}$$

$$y = \frac{3}{2}x - \frac{5}{2}$$

$$x = \frac{2}{3}y - \frac{5}{3}$$

$$\left[x + \frac{5}{3} = \frac{2}{3}y \right]$$

$$\frac{2}{3}x + \frac{5}{3} = y$$

$$h^{-1}(x) = \frac{2}{3}x + \frac{5}{3}$$

$$\begin{aligned} h \circ h^{-1}(x) &= \frac{3}{2} \left(\frac{2}{3}x + \frac{5}{3} \right) - \frac{5}{2} \\ &= x + \frac{5}{2} - \frac{5}{2} \\ &= x \end{aligned}$$

$$\begin{aligned} h^{-1} \circ h(x) &= \frac{2}{3} \left(\frac{3}{2}x - \frac{5}{2} \right) + \frac{5}{3} \\ &= x - \frac{5}{3} + \frac{5}{3} \\ &= x \end{aligned}$$

$$52) a) p = 84x + 60000$$

$$b) \begin{array}{r} x = 84p + 60000 \\ -60000 \quad -60000 \\ \hline x - 60000 = 84p \\ \hline \frac{x - 60000}{84} = \frac{84p}{84} \end{array}$$

$$p = \frac{x - 60000}{84}$$

switch x & p back

$$x = \frac{p - 60000}{84}$$

$$c) \begin{array}{r} x = \frac{180000 - 60000}{84} \end{array}$$

$$x = \frac{120000}{84} = 1429 \text{ ft}^2$$