6.4- Properties of Logarithmic Functions

Product and Quotient Properties of Logarithms

For $m > 0$, $n > 0$, $b > 0$, and $b \neq 1$:

Product Property $\log_b(mn) = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Given that $\log_2 3 = 1.5850$, approximate each expression below by using the Product and Quotient Properties of Logarithms

a. $\log_2 18$

$\log_2(2 \cdot 9)$

$\log_2(2 \cdot 3 \cdot 3)$

$\log_2 2 + \log_2 3 + \log_2 3$

$2^x = 2$

$1 + 1.5850 + 1.5850$

4.1700

b. $\log_2 \frac{3}{4}$

$\log_2(2 \cdot 2)$

$\log_2 2 + \log_2 2$

$1 + 1$

$\log_2 3 - \log_2 4$

$2^x = 4$

$1.5850 - 2$

$-.4150$

Write each expression as a single logarithm. Then simplify if possible.

a. $\log_4 18 - \log_4 6$

$\log_4 \frac{18}{6}$

$\log_4 3$

b. $\log_b 4x - \log_b 3y + \log_b y$

$\log_b \frac{4x}{3y} + \log_b y$

$\log_b \frac{4xy}{3y}$

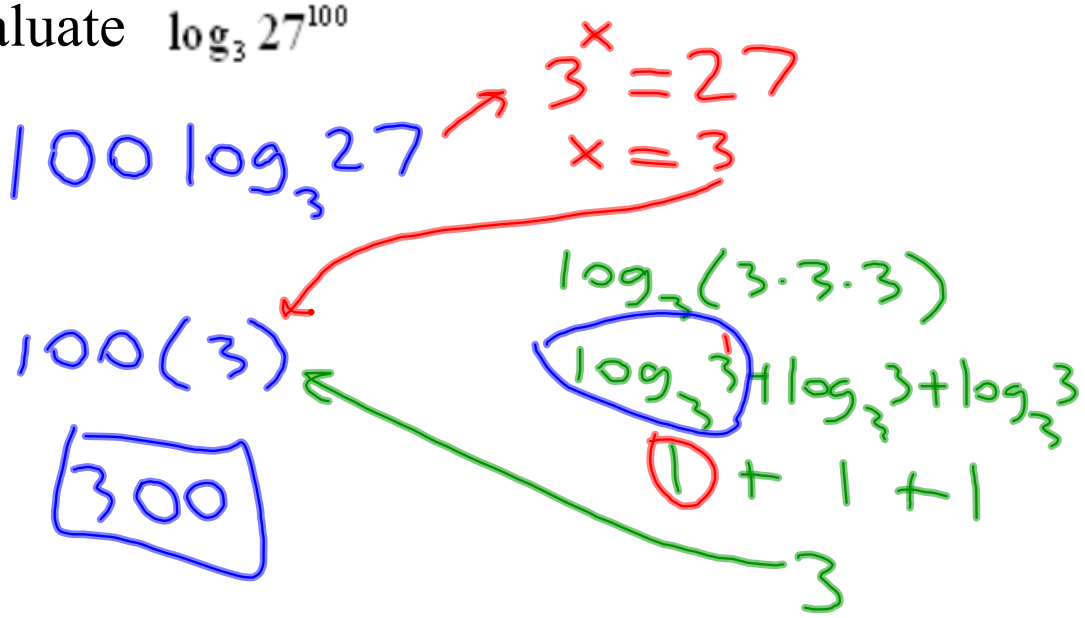
$\log_b \frac{4x}{3}$

Power Property of Logarithms

For $m > 0$, $b > 0$, $b \neq 1$, and any real number p:

$$\log_b m^p = p \log_b m$$

Evaluate $\log_3 27^{100}$



Exponential-Logarithmic Inverse Properties

For $b > 0$ and $b \neq 1$:

$$\log_b b^x = x \text{ and } b^{\log_b x} = x \text{ for } x > 0$$

Evaluate each expression.

a. $7^{\log_7 11} - \log_3 81$

$\log_3(3 \cdot 3 \cdot 3 \cdot 3) \rightarrow \log_3 3^4$

$3^x = 81$

$x = 4$

$11 - 4$

7

b. $\log_8 8^5 + 3^{\log_3 8}$

$5 + 8$

13

One-to-One Property of Logarithm

If $\log_b x = \log_b y$, then $x = y$

Homework

Pg. 382-383 #14-22 even, 38-48 even, 58-62 even, 72, 74

14) $\log_2 8xy$

$2^x = 8 \rightarrow x = 3$

$\log_2 8 + \log_2 x + \log_2 y$

$\log_2(2 \cdot 2 \cdot 2)$

$\log_2 2 + \log_2 2 + \log_2 2$

$1 + 1 + 1$

$3 + \log_2 x + \log_2 y$

16) $\log_4 \frac{x}{32}$

$\log_4 x - \log_4 32$

$\log_4 x - \log_4 (4 \cdot 4 \cdot 2)$

$\log_4 x - (\log_4 4 + \log_4 4 + \log_4 2)$

$\log_4 x - (1 + 1 + \frac{1}{2})$

$\log_4 x - \frac{5}{2}$

18) $\log_2 35$

$\log_2(7 \cdot 5)$

$\log_2 7 + \log_2 5$

$2.8074 + 2.3219$

5.1293

22) $\log_2 105$

$\log_2(7 \cdot 5 \cdot 3)$

$\log_2 7 + \log_2 5 + \log_2 3$

$2.8074 + 2.3219 + 1.5850$

$\boxed{6.7143}$

38) $7\log_3 x - 4\log_3 x$

$\log_3 y^7 - \log_3 x^4$

$\log_3 \frac{y^7}{x^4}$

40) $\frac{1}{2}\log_b 3c + \frac{1}{2}\log_b 4d - 2\log_b 5e$

$\log_b (3c)^{1/2} + \log_b (4d)^{1/2} - \log_b (5e)^2$

$\log_b \frac{(3c)^{1/2}(4d)^{1/2}}{(5e)^2} = \log_b \frac{\sqrt{12cd}}{25e^2}$

42) $2 + 4\log_3 x$

$\log_3 9 + \log_3 x^4$

$\log_3 9x^4$

46) $\log_{10} 10^2$

2

48) $5^{\log_5 7} + \log_3 9$

$\log_3 3^2$

$7 + 2$

9

$3^x = 9$

$x = 2$

58) $\log_{10}(5x-3) - \log_{10}(x^2+1) = 0$

$+\log_{10}(x^2+1) \longrightarrow$

$\log_{10}(5x-3) = \log_{10}(x^2+1)$

$5x-3 = x^2+1$

$-5x+3 \quad -5x+3$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4, 1$

58) $\log_{10}(5x-3) - \log_{10}(x^2+1) = 0$

$$\log_{10}\frac{5x-3}{x^2+1} = 0$$

$$10^0 = \frac{5x-3}{x^2+1}$$

$$\frac{1}{1} = \frac{5x-3}{x^2+1}$$

$$x^2+1 = 5x-3$$

$$-5x+3 \quad -5x+3$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x = 4, 1$$

60) $\log_b(x^2-2)+2\log_b 6 = \log_b 6x$

$\log_b(x^2-2)+\log_b 6^2 = \log_b 6x$

$\log_b 36(x^2-2) = \log_b 6x$

$\log_b(36x^2-72) = \log_b 6x$

$36x^2-72 = 6x$

$-6x \quad -6x$

$\frac{36x^2}{6} - \frac{6x}{6} - \frac{72}{6} = \frac{0}{6}$

$6x^2 - x - 12 = 0$

$(3x+4)(2x-3) = 0$

$x = -\frac{4}{3}, \frac{3}{2}$

b2) $\log_5 2 + 2\log_5 T = \log_5(3-T)$

$\log_5 2 + \log_5 T^2 = \log_5(3-T)$

$\log_5 2T^2 = \log_5(3-T)$

$2T^2 = 3-T$

$+T-3-3+T$

$2T^2+T-3=0$

$(2T+3)(T-1)=0$

$T=-\frac{3}{2}, \boxed{1}$

72) $\log_4(\log_3 x) = 0$

$x = 3$

$4^0 = \log_3 x$

$1 = \log_3 x$

$3^1 = x$

74) $\log_{10} S = .425 \log_{10} W + .725 \log_{10} H + \log_{10} 71.84$

$-\log_{10} S \qquad\qquad -\log_{10} S$

$$0 = \log_{10} W^{.425} + \log_{10} H^{.725} + \log_{10} 71.84 - \log_{10} S$$

$$0 = \log_{10} \frac{W^{.425} H^{.725} (71.84)}{S}$$

$$S\left[1 = 10^{0} = \frac{71.84\, W^{.425} H^{.725}}{S}\right]$$

$$S = 71.84\, W^{.425} H^{.725}$$

74) $\log_{10} S = .425 \log_{10} W + .725 \log_{10} H + \log_{10} 71.84$

$\log_{10} S = \log_{10} W^{.425} + \log_{10} H^{.725} + \log_{10} 71.84$

$\log_{10} S = \log_{10} W^{.425} H^{.725} (71.84)$

$S = 71.84 W^{.425} H^{.725}$