7.5- Zeros of Polynomial Functions

Rational Root Theorem

Let P be a polynomial function with integer coefficients in standard form. If $\frac{p}{q}$ (in lowest terms) is a root of $P(x) = 0$, then

* p is a factor of the constant term P and
* q is a factor of the leading coefficient of P.

Example of polynomial P: $P(x) = qx^3 + \ldots + p$

Find all of the rational roots of $3x^3 - 17x^2 + 59x - 65 = 0$

Factors of -65: $\pm 1, \pm 5, \pm 13, \pm 65$

Factors of 3: $\pm 1, \pm 3$

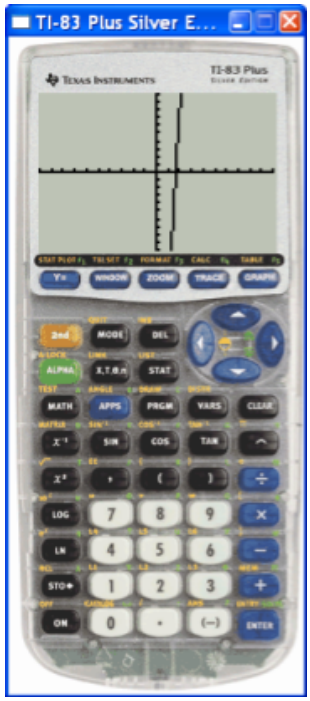
$\pm \frac{1}{1}, \pm \frac{1}{3}$ $\quad \pm \frac{5}{1}, \pm \frac{5}{3}$ $\quad \pm \frac{13}{1}, \pm \frac{13}{3}$ $\quad \pm \frac{65}{1}, \pm \frac{65}{3}$

$\frac{5}{3}$ | 3 −17 59 −65

5 −20 65

3 −12 39 | 0

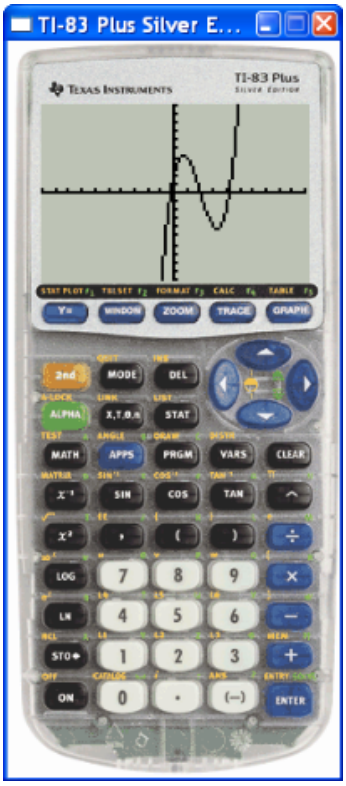
$x = \frac{5}{3}$



Complex Conjugate Root Theorem

If P is a polynomial function with real-number coefficients and $a + bi$ (where $b \neq 0$) is a root of $P(x) = 0$, then $a - bi$ is also a root of $P(x) = 0$.

Find all of the zeros of $P(x) = x^3 - 6x^2 + 7x + 2$



Factors of 2 : $\pm 1, \pm 2$

Factors of 1 : ± 1

$\pm\frac{1}{1}$ $\pm\frac{2}{1}$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 7 & 2 \\ & & 2 & -8 & -2 \\ \hline & 1 & -4 & -1 & 0 \end{array}$$

$x^2 - 4x - 1$

$a = 1$, $b = -4$, $c = -1$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16+4}}{2}$$

$$x = \frac{4 \pm \sqrt{20}}{2}$$

$$x = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$x = 2, 2 \pm \sqrt{5}$

Write a polynomial function, *P*, in factored form and in standard form by using the given information.

P is of degree 3; $P(0) = 20$; zeros: -2, 1, 2

$$P(x) = 5(x+2)(x-1)(x-2)$$

$$= 5(x^2 - x + 2x - 2)(x-2)$$

$$= 5(x^2 + x - 2)(x-2)$$

$$= 5(x^3 - 2x^2 + x^2 - 2x - 2x + 4)$$

$$= 5(x^3 - x^2 - 4x + 4)$$

$$P(x) = 5x^3 - 5x^2 - 20x + 20$$

Homework

Pg. 463-464 #16-26 even, 44, 46, 52

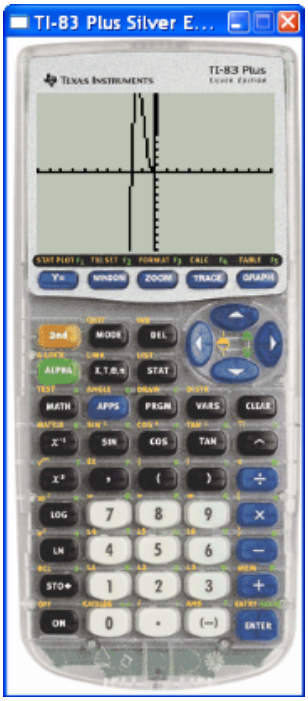
16) $15a^3 + 38a^2 + 17a + 2 = 0$

factors of 2: $\pm 1, \pm 2$

factors of 15: $\pm 1, \pm 3, \pm 5, \pm 15$

$\pm \frac{1}{1}, \pm \frac{1}{3}, \pm \frac{1}{5}, \pm \frac{1}{15}$

$\pm \frac{2}{1}, \pm \frac{2}{3}, \pm \frac{2}{5}, \pm \frac{2}{15}$



$-2 \;|\; 15 \quad 38 \quad 17 \quad 2$

$\qquad\quad -30 \quad -16 \quad -2$

$15 \quad 8 \quad 1 \quad |0$

$15a^2 + 8a + 1 = 0$

$(3a+1)(5a+1) = 0$

$X = -2, -\frac{1}{3}, -\frac{1}{5}$

18) $18c^3+9c^2-23c+6=0$

Factors of 6: $\pm1, \pm2, \pm3, \pm6$

Factors of 18: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{1}{9}, \pm\frac{1}{18}$

$\pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{3}, \pm\frac{2}{6}, \pm\frac{2}{9}, \pm\frac{2}{18}$

$\pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{3}{3}, \pm\frac{3}{6}, \pm\frac{3}{9}, \pm\frac{3}{18}$

$\pm\frac{6}{1}, \pm\frac{6}{2}, \pm\frac{6}{3}, \pm\frac{6}{6}, \pm\frac{6}{9}, \pm\frac{6}{18}$

$-\frac{3}{2}$ | 18 9 -23 6

-27 27 -6

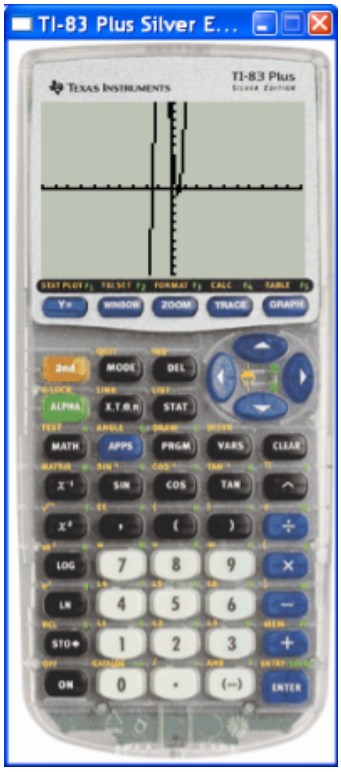
18 -18 4 | 0

$\frac{18x^2}{2}-\frac{18x}{2}+\frac{4}{2}=\frac{0}{2}$

$9x^2-9x+2=0$

$(3x-1)(3x-2)=0$

$x=-\frac{3}{2}, \frac{1}{3}, \frac{2}{3}$



20) $18x^4+15x^3-34x^2+15x-2=0$

Factors of 2: $\pm1, \pm2$

Factors of 18: $\pm1, \pm3, \pm6, \pm9, \pm18$

$\pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{1}{9}, \pm\frac{1}{18}$ $\pm\frac{2}{1}, \pm\frac{2}{3}, \pm\frac{2}{6}, \pm\frac{2}{9}, \pm\frac{2}{18}$

-2 | 18 15 -34 15 -2

-36 42 -16 2

18 -21 8 -1 | 0

$\frac{1}{2}$ | 18 -21 8 -1

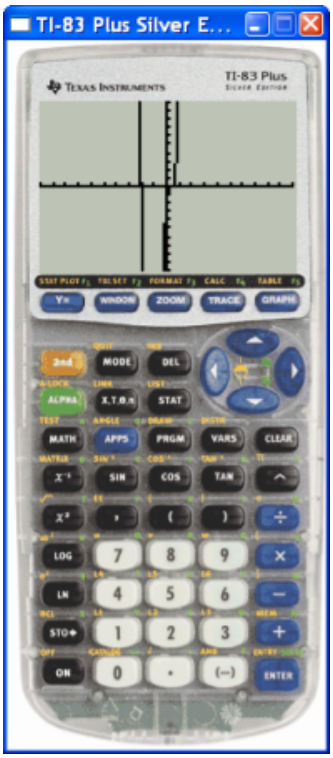
9 -6 1

18 -12 2 | 0

$\frac{18x^2}{2}-\frac{12x}{2}+\frac{2}{2}=\frac{0}{2}$

$9x^2-6x+1=0$

$(3x-1)(3x-1)=0$



$x=-2, \frac{1}{3}, \frac{1}{2}$

24) $f(x) = 4x^3 - 20x^2 - 3x + 15$

$$\begin{array}{r|rrrr} 5 & 4 & -20 & -3 & 15 \\ & & 20 & 0 & -15 \\ \hline & 4 & 0 & -3 & 0 \end{array}$$

$4x^2 - 3 = 0 \rightarrow \frac{4x^2}{4} = \frac{3}{4}$

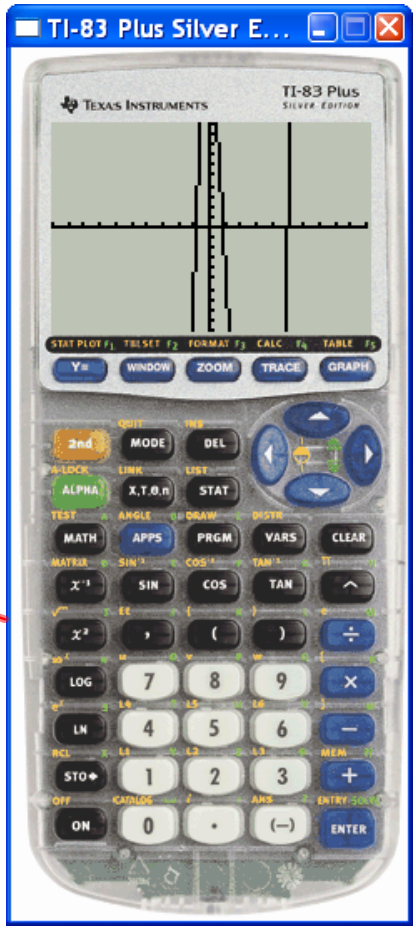
$x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

$(2x + \sqrt{3})(2x - \sqrt{3}) = 0$

$\pm\frac{\sqrt{12}}{8} = \pm\frac{\sqrt{4}\sqrt{3}}{8} = \pm\frac{2\sqrt{3}}{8} = \pm\frac{\sqrt{3}}{4}$

$\boxed{x = 5, \pm\frac{\sqrt{3}}{2}}$



26) $x^3 - 2x^2 - 35x + 78$

factors of 78: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 13, \pm 26, \pm 39, \pm 78$

factors of 1: ± 1

$\pm\frac{1}{1}$ $\pm\frac{2}{1}$ $\pm\frac{3}{1}$ $\pm\frac{6}{1}$ $\pm\frac{13}{1}$ $\pm\frac{26}{1}$

$\pm\frac{39}{1}$ $\pm\frac{78}{1}$

$-6 \mid 1 \quad -2 \quad -35 \quad 78$

$\quad\quad -6 \quad 48 \quad -78$

$1 \quad -8 \quad 13 \quad 0$

$x^2 - 8x + 13$

$a = 1$

$b = -8$

$c = 13$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{12}}{2}$$

$$x = -6, \frac{8 \pm \sqrt{12}}{2}$$

44) P is of degree 3; $P(0)=24$; zeros: $-1, 2, 4$

$$P(x) = 3(x+1)(x-2)(x-4)$$

$$= 3(x^2-2x+x-2)(x-4)$$

$$= 3(x^2-x-2)(x-4)$$

$$= 3(x^3-4x^2-x^2+4x-2x+8)$$

$$= 3(x^3-5x^2+2x+8)$$

$$P(x) = 3x^3-15x^2+6x+24$$

46) P is of degree 5; $P(0)=2$; zeros: 1 (mult 3), 2 (mult 2)

$$P(x) = -\tfrac{1}{2}(x-1)(x-1)(x-1)(x-2)(x-2)$$

$$= (x^2-2x+1)(x-1)(x^2-4x+4)$$

$$= (x^3-x^2-2x^2+2x+x-1)(x^2-4x+4)$$

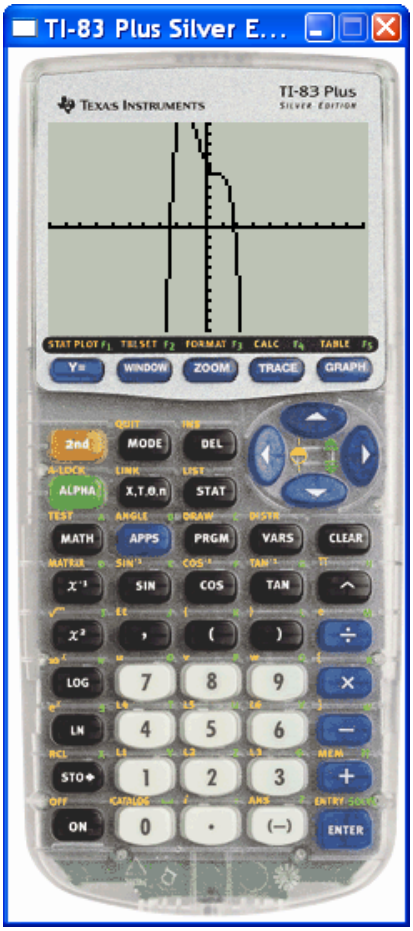
$$= (x^3-3x^2+3x-1)(x^2-4x+4)$$

$$= (x^5-4x^4+4x^3-3x^4+12x^3-12x^2+3x^3 - 12x^2+12x-x^2+4x-4)$$

$$= -\tfrac{1}{2}(x^5-7x^4+19x^3-25x^2+16x-4)$$

$$P(x) = -\frac{x^5}{2}+\frac{7x^4}{2}-\frac{19x^3}{2}+\frac{25x^2}{2}-8x+2$$

52) $-x^4 + 3x^2 - 3x + 6$



$x = -2.3, 1.8$

b) 12.2

c) **262.2**