8.1- Inverse, Joint, and Combined Variation

Inverse Variation

Two variables, x and y, have an **inverse-variation** relationship if there is a nonzero number k such that $xy = k$, or $y = \frac{k}{x}$. The **constant of variation** is k.

Example: The variable y varies inversely as x, and $y = 120$ when $x = 6.5$. Find the constant of variation and write an equation for the relationship. Then find y when x is 1.5, 4.5, 8, 12.5, and 14.

$y = \frac{k}{x}$

$6.5[120 = \frac{k}{6.5}]$

$780 = k$

$y = \frac{780}{x}$

Joint Variation

If $y = kxz$, then y **varies jointly** as x and z, and the constant of variation is k.

Example:

a. Write an equation for the volume of a rectangular prism whose base has a length of 12 inches. Identify the type of variation and the constant of variation.

$V = 12wh$

Joint variation

$k = 12$

b. Find the volume of the prism if the width of the base is 2 inches and the height of the prism is 4 inches.

$V = (12\text{ in})(2\text{ in.})(4\text{ in.})$

$V = 96\text{ in.}^3$

Combined Variation

Combined variation occurs when more than one type of variation occurs in the same equation.

Example: y varies inversely as x and z. Write the variation equation. Identify the type of variation.

$$y = \frac{k}{xz}$$

Combined Variation

NOTES:
The rotational speeds, S_A and S_B, of Gear A with t_A teeth and Gear B with t_B teeth are related as indicated below.

$$t_A S_A = t_B S_B$$

The Combined Variation Equation for the Rotational Speed of Gear B in terms of t_A, S_A, and t_B is as follows:

$$S_B = \frac{t_A S_A}{t_B}$$

Example:
A bicycle's pedal gear has 46 teeth and is rotating at 55 revolutions per minute. If the pedal gear is linked to a rear-wheel gear that has 24 teeth and is attached to a 27-inch wheel, at what speed, in miles per hour, is the bicycle travelling?

$$S_B = \frac{46(55)\text{ rev}}{24\ \text{min}} \cdot \frac{60\ \text{min}}{1\ \text{hr}} \cdot \frac{27\pi\ \text{in}}{1} \cdot \frac{1\ \text{ft}}{12\ \text{in}} \cdot \frac{1\ \text{mi}}{5280\ \text{ft}} \approx 85\ \text{MPH}$$

Homework

pg. 486-487 #16, 18, 24, 26, 32, 34, 38, 40, 43-46 all

16) $y = .25$ when $x = .3$

$y = \frac{k}{x}$

$.3\left[.25 = \frac{k}{.3}\right]$

$k = .075$

$y = \frac{.075}{x}$

x	y
.1	.75
.2	.375
.3	.25
.4	.1875

24) $y = kxz$

$.5 = k(10)(3)$

$\frac{.5}{30} = \frac{30k}{30}$

$\frac{1}{2} \cdot \frac{1}{30} = k$

$\frac{1}{60} = k$

$y = \frac{1}{60} xz$

$y = \frac{1}{\cancel{60}_{10}}(1.8)(\cancel{6})$

$y = .18$

32) $z = \frac{kxy}{w}$

$9\left[15 = \frac{k(3)(4)}{9}\right]$

$\frac{135}{12} = \frac{12k}{12}$

$k = 11.25$

$z = \frac{11.25xy}{w}$

$z = \frac{11.25(1.5)(20.5)}{5.4}$

$z = 64.0625$

38) $(3, y)$ and $(18, 6)$

$$3y = 18(6)$$

$$\frac{3y}{3} = \frac{108}{3}$$

$$y = 36$$

40) (x, x) and $(5, 125)$

$$\sqrt{x^2} = \sqrt{625}$$

$$x = \pm 25$$

43) $E = \frac{C}{D^2}$

$.01 = \frac{C}{6^2}$

$\frac{1}{2}\left[2(.01) = \frac{2C}{6^2}\right]$

$.01 = \frac{C}{6^2}$ → distance

44) $L = 5A^2$

$L = 5\ 250^2$

$L = 625005$

$3L = 25A^2$

$3(62500)5 = 25A^2$

$\frac{187,5005}{25} = \frac{25A^2}{25}$

$\sqrt{93750} = \sqrt{A^2}$

306.2 MPH = A

45) $h = \frac{k\,d\,A}{T}$

$9000 = \frac{k\,30(1500)}{.25}$

$\frac{9000}{180000} = \frac{k\,180000}{180000}$

$k = .05$

$h = \frac{.05\,d\,A}{T}$

$h = \frac{.05(15)(1500)}{.2}$

$h = 5625 \frac{cal.}{hr.}$

46) $B = \frac{K}{L}$

a) $24.1\left[2.8 = \frac{K}{24.1}\right]$ $K = 67.5$

b) $B = \frac{67.5}{13.2} = 5.1 \frac{\text{beats}}{\text{sec}}$

c) $L\left[1.9 = \frac{67.5}{L}\right] \rightarrow \frac{1.9L}{1.9} = \frac{67.5}{1.9} \rightarrow L = 35.5\text{cm}$