

8.4- Adding and Subtracting Rational Expressions

Adding and Subtracting Rational Expressions

To add or subtract two rational expressions, find a common denominator, rewrite each expression by using the common denominator, and then add or subtract. Simplify the resulting rational expression

$$\text{Simplify } \frac{3x-1}{2x-1} + \frac{5+2x}{2x-1} = \boxed{\frac{5x+4}{2x-1}}$$

$$\text{Simplify } \frac{2x}{x-5} - \frac{10}{x-5} = \frac{2x-10}{x-5} = \frac{2(x-5)}{\cancel{x-5}} = \boxed{2}$$

Simplify $\frac{x}{x+5} + \frac{-50}{x^2-25}$

$$\frac{\cancel{x-5}}{\cancel{x-5}} \cdot \frac{x}{x+5} + \frac{-50}{(x+5)(x-5)}$$

$$\frac{x^2-5x}{(x+5)(x-5)} + \frac{-50}{(x+5)(x-5)}$$

$$\frac{x^2-5x-50}{(x+5)(x-5)} = \frac{(x-10)\cancel{(x+5)}}{\cancel{(x+5)}(x-5)} = \boxed{\frac{x-10}{x-5}}$$

Simplify $\frac{6}{x^2 - 2x} - \frac{1}{x^2 - 4}$

$$\frac{\cancel{x+2}}{\cancel{x+2}} \cdot \frac{6}{x(x-2)} - \frac{1}{(x+2)(x-2)} \cdot \frac{\cancel{x}}{\cancel{x}}$$

$$\frac{6x+12}{x(x+2)(x-2)} - \frac{x}{x(x+2)(x-2)}$$

$$\frac{5x+12}{x(x+2)(x-2)} = \frac{5x+12}{x(x^2-4)} = \boxed{\frac{5x+12}{x^3-4x}}$$

Simplify $\frac{a}{\frac{1}{a} - \frac{1}{a}} - \frac{a}{\frac{1}{a} + \frac{1}{a}}$

$$\frac{a}{a} \cdot \frac{a}{1} - \frac{a}{a} \cdot \frac{a}{1} + \frac{1}{a}$$

$$\frac{\frac{a}{1}}{\frac{a^2}{a} - \frac{1}{a}} - \frac{\frac{a}{1}}{\frac{a^2}{a} + \frac{1}{a}} = \frac{\frac{a}{1}}{a^2 - 1} - \frac{\frac{a}{1}}{a^2 + 1}$$

$$\frac{1}{a} \cdot \frac{a}{a^2 - 1} - \frac{1}{a} \cdot \frac{a}{a^2 + 1} = \frac{a^2 + 1}{a^2 + 1} \cdot \frac{a^2}{a^2 - 1} - \frac{a^2}{a^2 + 1} \cdot \frac{a^2 - 1}{a^2 - 1}$$

$$\frac{a^4 + a^2}{(a^2 + 1)(a^2 - 1)} - \frac{a^4 - a^2}{(a^2 + 1)(a^2 - 1)} = \frac{a^4 + a^2 - a^4 + a^2}{(a^2 + 1)(a^2 - 1)}$$

$$\frac{2a^2}{(a^2 + 1)(a^2 - 1)} = \boxed{\frac{2a^2}{a^4 - 1}}$$

Homework

Pg. 509-510 #28-44 even, 47-50 all

$$28) \frac{\frac{4}{x+2}}{\frac{x+2}{3}} - \frac{3}{x+2}$$

$$\frac{4}{x+2} \cdot \frac{3}{x+2} - \frac{3}{x+2}$$

$$\frac{12}{(x+2)(x+2)} - \frac{3}{x+2} \cdot \frac{x+2}{x+2}$$

$$\frac{12 - 3x - 6}{(x+2)(x+2)} = \frac{-3x + 6}{(x+2)(x+2)} = \frac{-3x + 6}{x^2 + 4x + 4}$$

$$30) \frac{\frac{2x+10}{x-1}}{\frac{x+5}{x^2-1}} - \frac{4}{x+1}$$

$$\frac{2(x+5)}{\cancel{x-1}} \cdot \frac{(x+1)\cancel{(x-1)}}{\cancel{x+5}} - \frac{4}{x+1}$$

$$\frac{x+1}{x+1} \cdot \frac{2x+2}{1} - \frac{4}{x+1} = \frac{2x^2+4x+2}{x+1} - \frac{4}{x+1}$$

$$\frac{2x^2+4x-2}{x+1}$$

$$32 \quad \frac{x-y}{x-y}$$

$$\frac{x-y}{\frac{x}{y} \cdot \frac{1}{x} - \frac{1}{y} \cdot \frac{x}{x}} = \frac{x-y}{\frac{y-x}{xy}}$$

$$\frac{x-y}{1} \cdot \frac{xy}{y-x} = \frac{x-y}{1} \cdot \frac{xy}{-x+y} = \frac{\cancel{x-y}}{1} \cdot \frac{xy}{-1(\cancel{x-y})}$$

$$\boxed{-xy}$$

36)

$$\begin{aligned} & \frac{\frac{x-7}{x-7} \cdot 7}{x+7} + \frac{\frac{x+7}{x+7} \cdot (-x)}{x-7} + \frac{2x}{x^2-49} \\ & \frac{7x-49}{(x-7)(x+7)} + \frac{-x^2-7x}{(x-7)(x+7)} + \frac{2x}{(x-7)(x+7)} \\ & \frac{-x^2+2x-49}{(x-7)(x+7)} = \boxed{\frac{-x^2+2x-49}{x^2-49}} \end{aligned}$$

$$38) (a-b)^{-1} - (a+b)^{-1}$$

$$\frac{a+b}{a+b} \cdot \frac{1}{a-b} - \frac{1}{a+b} \cdot \frac{a-b}{a-b}$$

$$\frac{a+b}{(a+b)(a-b)} - \frac{a-b}{(a+b)(a-b)}$$

$$\frac{a+b-a+b}{(a+b)(a-b)} = \frac{2b}{a^2-b^2}$$

$$40) \frac{x}{x-y} - \frac{x^2+y^2}{x^2-y^2} + \frac{y}{x+y}$$

$$\frac{x+y}{x+y} \cdot \frac{x}{x-y} - \frac{x^2+y^2}{(x-y)(x+y)} + \frac{y}{x+y} \cdot \frac{x-y}{x-y}$$

$$\frac{x^2+xy}{(x+y)(x-y)} - \frac{x^2+y^2}{(x+y)(x-y)} + \frac{xy-y^2}{(x+y)(x-y)}$$

$$\frac{\cancel{x^2} + xy - \cancel{x^2} - y^2 + xy - y^2}{(x+y)(x-y)}$$

$$\frac{2xy - 2y^2}{(x+y)(x-y)} = \frac{2y(\cancel{x-y})}{(x+y)(\cancel{x-y})}$$

$$\boxed{\frac{2y}{x+y}}$$

$$42) \quad \frac{3x+2}{x-5} = \frac{Ax+D}{x-5}$$

$$A=3$$

$$D=2$$

$$44) \quad \frac{2x^2+5}{x^2+11x+30} = \frac{Ax}{x+5} + \frac{Cx+D}{x+6}$$

$$\frac{2x^2+5}{(x+5)(x+6)} = \frac{\overset{x+6}{x+6} \cdot \frac{Ax}{x+5}}{\cancel{x+6}} + \frac{Cx+D}{x+6} \cdot \frac{\overset{x+5}{x+5}}{\cancel{x+5}}$$

$$= \frac{Ax^2 + 6Ax + Cx^2 + 5(Cx+D) + 5D}{(x+5)(x+6)}$$

$$\frac{2x^2+5}{(x+5)(x+6)} = \frac{(A+C)x^2 + (6A+5C+D)x + 5D}{(x+5)(x+6)}$$

$$A+C=2$$

$$A=2-C$$

$$6A+5C+D=0$$

$$\begin{array}{l} D=1 \\ A=-11 \\ C=13 \end{array}$$

$$6(2-C)+5C+1=0$$

$$12-6C+5C+1=0$$

$$-C+13=0$$

$$\begin{array}{r} +C \qquad +C \\ \hline \end{array}$$

$$C=13$$

$$47) R(T) = \frac{1}{\frac{1}{R(a)} + \frac{1}{R(b)} + \frac{1}{R(c)}}$$

$$R(T) = \frac{1}{\frac{24}{24} \cdot \frac{1}{5} + \frac{15}{15} \cdot \frac{1}{8} + \frac{1}{12} \cdot \frac{10}{10}}$$

$$= \frac{1}{\frac{24 + 15 + 10}{120}} = \frac{1}{\frac{49}{120}} = \boxed{\frac{120}{49}}$$

$$47) b. R(T) = \frac{1}{\frac{1}{R(a)} + \frac{1}{R(b)} + \frac{1}{R(c)}}$$

$$\frac{1}{\frac{R_b R_c}{R_b R_c} \cdot \frac{1}{R_a} + \frac{R_a R_c}{R_a R_c} \cdot \frac{1}{R_b} + \frac{1}{R_c} \cdot \frac{R_a R_b}{R_a R_b}}$$

$$\frac{1}{\frac{R_b R_c + R_a R_c + R_a R_b}{R_a R_b R_c}} = \frac{R_a R_b R_c}{R_b R_c + R_a R_c + R_a R_b}$$