Algebra II\ Trig Ch. 8: 8.1-8.4 Review

Solve problems involving inverse, joint, or combined variation.

1. A variable *a* varies directly as *b* and inversely as the square of *c*. If *a* = 3, *b* = 18, and *c* = 2, what is *a* when *b* = 20 and *c* = 6?

$a = \frac{kb}{c^2}$

$4\left[3 = \frac{k(18)}{2^2}\right]$

$\frac{12}{18} = \frac{18k}{18}$

$\frac{2}{3} = k$

$a = \frac{\frac{2}{3}b}{c^2} = \frac{\frac{2}{3}(20)}{6^2}$

$= \frac{\frac{40}{3}}{36}$

$\boxed{a = .37}$

Identify the domain, asymptotes, and holes in the graph of each rational function.

2. $f(x) = \dfrac{2x^3 + 6x^2}{x^2 - x - 12} = \dfrac{2x^2(x+3)}{(x-4)(x+3)}$

vert. asym – $x = 4$

horiz. asym – None

holes – $x = -3$

domain – $x \neq -3, 4$

3. $g(x) = \frac{3x^2 + x - 4}{x^2 + 2x - 3} = \frac{(3x+4)(x-1)}{(x+3)(x-1)}$

vert. asym – $x = -3$

horiz. asym – $y = 3$

holes – $x = 1$

domain – $x \neq -3, 1$

4. $h(x) = \frac{2x}{x^2 - x - 2} = \frac{2x}{(x-2)(x+1)}$

vert. asym – $x = 2, -1$

horiz. asym – $y = 0$

holes – None

domain – $x \neq 2, -1$

Simplify each expression

5. $\dfrac{\cancel{7}x(\cancel{12}x^5)}{\cancel{3}x^2(\cancel{28}x^2)} = \dfrac{x^{\cancel{5}\,2}}{\cancel{x^{3}}} = \boxed{x^2} = \dfrac{\cancel{x}\cancel{x}\cancel{x}\cancel{x}xx}{\cancel{x}\cancel{x}\cancel{x}\cancel{x}}$

$\dfrac{\cancel{84}x^{\cancel{6}\,2}}{\cancel{84}\cancel{x^4}} = x^2$

6. $\dfrac{x^2-8x-20}{12x-x^2-20} = \dfrac{(x-10)(x+2)}{-(x^2-12x+20)}$

$$= \frac{\cancel{(x-10)}(x+2)}{-\cancel{(x-10)}(x-2)}$$

$$= \boxed{\frac{x+2}{-x+2}}$$

7. $\dfrac{3x^2+10x-8}{3x^2-17x+10} \bullet \dfrac{5+9x-2x^2}{x^2+3x-4}$

$$\frac{(3x-2)(x+4)}{(3x-2)(x-5)} \cdot \frac{-(2x^2-9x-5)}{(x+4)(x-1)}$$

$$\frac{\cancel{(3x-2)}\cancel{(x+4)}}{\cancel{(3x-2)}\cancel{(x-5)}} \cdot \frac{-(2x+1)\cancel{(x-5)}}{\cancel{(x+4)}(x-1)}$$

$$\boxed{\frac{-2x-1}{x-1}}$$

8. $\dfrac{2x^2-x-1}{3x^2-2x-1}\bullet\dfrac{15x^3+5x^2}{4x^2-1}$ $= \dfrac{\cancel{(2x+1)}\cancel{(x-1)}}{\cancel{(3x+1)}\cancel{(x-1)}}\cdot\dfrac{5x^2\cancel{(3x+1)}}{\cancel{(2x+1)}(2x-1)}$

$$\boxed{\dfrac{5x^2}{2x-1}}$$

9. $\dfrac{x^2-2x-3}{x^2+x-20} \div \dfrac{x^2+2x+1}{x^2+6x+5}$

$$\frac{(x-3)\cancel{(x+1)}}{\cancel{(x+5)}(x-4)} \cdot \frac{\cancel{(x+5)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x+1)}}$$

$$\boxed{\frac{x-3}{x-4}}$$

10. $\dfrac{x+4}{x^2-9} \div \dfrac{\dfrac{x^2+4x}{x+3}}{\dfrac{x-3}{x}} = \dfrac{x+4}{x^2-9} \div \left(\dfrac{x^2+4x}{x+3} \div \dfrac{x-3}{x}\right)$

$\dfrac{x+4}{(x-3)(x+3)} \div \left(\dfrac{x(x+4)}{(x+3)} \cdot \dfrac{x}{(x-3)}\right)$

$\dfrac{\cancel{x+4}}{\cancel{(x-3)}\cancel{(x+3)}} \cdot \dfrac{\cancel{(x+3)}\cancel{(x-3)}}{x^2\cancel{(x+4)}} = \boxed{\dfrac{1}{x^2}}$

11. $\frac{2x+1}{3x-4}+\frac{x-1}{3x-4} = \boxed{\frac{3x}{3x-4}}$

12. $\dfrac{2x-1}{x+8}+\dfrac{3x}{x^2-64} = \dfrac{(x-8)}{(x-8)} \cdot \dfrac{2x-1}{x+8} + \dfrac{3x}{(x-8)(x+8)}$

$$\frac{2x^2-17x+8+3x}{(x-8)(x+8)}$$

$$\boxed{\frac{2x^2-14x+8}{x^2-64}}$$

13. $\frac{-2x-3}{x^2-3x}+\frac{+x}{x-3} = \frac{-2x-3}{x(x-3)} + \frac{x}{x-3}\cdot\frac{\cancel{x}}{x}$

$$\frac{x^2-2x-3}{x(x-3)} = \frac{(x+1)\cancel{(x-3)}}{x\cancel{(x-3)}}$$

$$\boxed{\frac{x+1}{x}}$$

14. $\dfrac{5}{\dfrac{x}{3x+1}} - \dfrac{7x^2+3x}{x}$ $= \dfrac{5}{1} \cdot \dfrac{3x+1}{x} + \dfrac{-7x^2 - 3x}{x}$

$$= \frac{15x+5-7x^2-3x}{x}$$

$$= \boxed{\frac{-7x^2+12x+5}{x}}$$

BONUS $$\frac{\dfrac{2}{x^2+2x-3}}{\dfrac{x-4}{x^3-x^2}}+\frac{\dfrac{6}{x^2+5x+6}}{\dfrac{x-4}{x+2}}$$