

# Solving Trigonometric Equations

## **Addition Property of Equality**

For any three algebraic expressions  $A$ ,  $B$ , and  $C$

$$\text{If } A = B$$

$$\text{then } A + C = B + C$$

Plainly: Adding the same quantity to both sides of an equation will not change the solution set.

## **Multiplication Property of Equality**

For any three algebraic expressions  $A$ ,  $B$ , and  $C$ , with  $C \neq 0$ ,

$$\begin{array}{l} \text{If } A = B \\ \text{then } AC = BC \end{array}$$

Plainly: Multiplying both sides of an equation by the same nonzero quantity will not change the solution set

Solve for  $x$ :  $2\sin x - 1 = 0$

$$\frac{2\sin x - 1 + 1 + 1}{2} = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

~~$x = 30^\circ$~~        ~~$x = 30^\circ, 150^\circ$~~

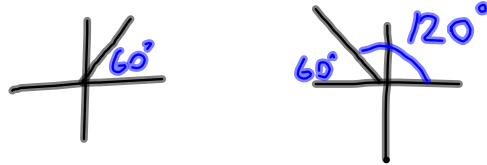
$$x = 30^\circ, 150^\circ$$

Solve  $2\sin \theta - 3 = 0$ , if  $0^\circ \leq \theta < 360^\circ$

$$\begin{array}{r} \phantom{2\sin\theta} - 3 \\ +3 \phantom{-3} \\ \hline 2\sin\theta = 3 \\ \phantom{2\sin\theta} \phantom{=} - 3 \\ \phantom{2\sin\theta} \phantom{=} +3 \\ \hline 2\sin\theta = 3 \\ \phantom{2\sin\theta} \phantom{=} - 3 \\ \phantom{2\sin\theta} \phantom{=} +3 \\ \hline \sin\theta = \frac{3}{2} \end{array}$$

No Solution

Find all degree solutions to  $\sin(2A - 50^\circ) = \frac{\sqrt{3}}{2}$



$$2A - 50^\circ = 60^\circ + 360^\circ n$$

$$+50^\circ \quad +50^\circ$$

$$\frac{2A}{2} = \frac{110^\circ}{2} + \frac{360^\circ n}{2}$$

$$A = 55^\circ + 180^\circ n$$

$$2A - 50^\circ = 120^\circ + 360^\circ n$$

$$+50^\circ \quad +50^\circ$$

$$\frac{2A}{2} = \frac{170^\circ}{2} + \frac{360^\circ n}{2}$$

$$A = 85^\circ + 180^\circ n$$

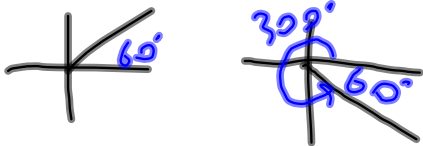


# **Homework**

**Problem Set 6.1 #2-54 even  
exclude 40-46 even**

$$2) \quad \frac{2 \cos \theta = \frac{1}{2}}{2}$$

$$\cos \theta = \frac{1}{2}$$

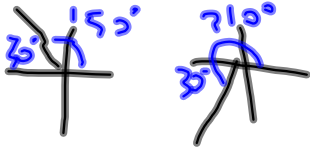


$$\theta = 60^\circ, 300^\circ$$

$$4) \quad 2 \cos \theta + \sqrt{3} = 0$$

$$\frac{-\sqrt{3} \quad -\sqrt{3}}{2}$$
$$\frac{2 \cos \theta = -\sqrt{3}}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$



$$\theta = 150^\circ, 210^\circ$$

$$b) \sqrt{3} \cot \theta - 1 = 0$$

$$\frac{\sqrt{3} \cot \theta}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\cot \theta = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \sqrt{3}$$

$$\cancel{60^\circ} \quad \cancel{60^\circ}$$

$$\theta = 60^\circ, 240^\circ$$

$$8) \sqrt{3} + 5 \sin T = 3 \sin T$$
$$\underline{\underline{- 5 \sin T - 5 \sin T}}$$

$$\underline{\underline{\frac{\sqrt{3}}{-2} = \frac{-2 \sin T}{-2}}}$$

$$\sin T = -\frac{\sqrt{3}}{2}$$

~~60°~~      ~~60°~~

$$T = 240^\circ, 300^\circ$$

$$T = \frac{4\pi}{3}, \frac{5\pi}{3}$$

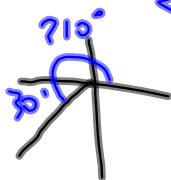
$$10) \quad \underbrace{5 \cos T + \sqrt{12}}_{-\cos T} = \underbrace{\cos T}_{-\cos T}$$

$$\underbrace{4 \cos T - \sqrt{12}}_{-\sqrt{12} \quad -\sqrt{12}} = 0$$

$$\frac{4 \cos T}{4} = \frac{-\sqrt{12}}{4}$$

$$\cos T = \frac{-\sqrt{12}}{4} = -\frac{\sqrt{4 \cdot 3}}{4} = -\frac{2\sqrt{3}}{4}$$

$$\cos T = -\frac{\sqrt{3}}{2}$$



$$T = 150^\circ, 210^\circ$$

$$T = \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$12) \quad 3 \sin T + 4 = 4$$

$$\underline{\quad -4 \quad -4 \quad}$$

$$\underline{\underline{3 \sin T = 0}}$$

$$\sin T = 0$$



$$T = 0, \pi$$

$$14) \quad 4 \sin \theta + 3 = 0$$

$$\frac{4 \sin \theta}{4} = \frac{-3}{4}$$

$$\sin \theta = -\frac{3}{4}$$

~~$\theta = 48.6^\circ$~~        ~~$\theta = 48.6^\circ$~~

$$\hat{\theta} = \sin^{-1} \frac{3}{4}$$

$$\theta = 228.6^\circ, 311.4^\circ$$

$$1b) \quad 4\cos\theta - 1 = 3\cos\theta + 4$$

$-3\cos\theta$                        $-3\cos\theta$

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$$\cos\theta - 1 = 4$$

$+1$      $+1$

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$$\cos\theta = 5$$

No Solution

$$20) (\cos x - 1)(2\cos x - 1) = 0$$

$$\cos x = 1$$

$$\cos x = \frac{1}{2}$$

~~$x = \frac{\pi}{2}$~~   ~~$x = \frac{3\pi}{2}$~~

$$x = 0^\circ, 60^\circ, 300^\circ$$
$$0, \frac{\pi}{3}, \frac{5\pi}{3}$$

$$22) \tan x (\tan x + 1) = 0$$

$$\tan x = 0$$



$$\tan x = -1$$



$$x = 0^\circ, 180^\circ, 135^\circ, 315^\circ$$

$$x = 0, \pi, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$24) \cos x - 2 \sin x \cos x = 0$$

$$\cos x (1 - 2 \sin x) = 0$$

$$\cos x = 0 \quad \sin x = \frac{1}{2}$$

$\downarrow$                        ~~$\downarrow$~~      ~~$\downarrow$~~

$$x = 90^\circ, 270^\circ, 30^\circ, 150^\circ$$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$$26) 2\cos^2 x + \cos x - 1 = 0$$

$$x = \cos x$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$x = \frac{1}{2}, -1$$

$$\cos x = \frac{1}{2}$$

~~$60^\circ$~~   ~~$300^\circ$~~

$$\cos x = -1$$

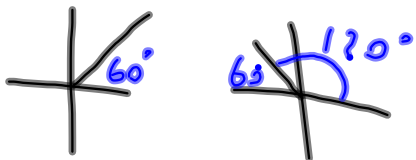
$180^\circ$

$$x = 60^\circ, 300^\circ, 180^\circ$$

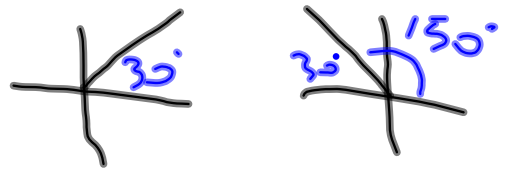
$$\frac{\pi}{3} \quad 5\frac{\pi}{3} \quad \pi$$

$$28) (2\sin\theta - \sqrt{3})(2\sin\theta - 1) = 0$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$



$$\sin\theta = \frac{1}{2}$$



$$\theta = 60^\circ, 120^\circ, 30^\circ, 150^\circ$$

$$30) \tan \theta - 2 \cos \theta \tan \theta = 0$$

$$\tan \theta (1 - 2 \cos \theta) = 0$$

$$\tan \theta = 0$$



$$\cos \theta = \frac{1}{2}$$

$$\cancel{60^\circ} \quad \cancel{300^\circ}$$

$$\theta = 0^\circ, 180^\circ, 60^\circ, 300^\circ$$

$$32) \quad 2 \sin^2 \theta - 7 \sin \theta = -3$$

$$\underline{\quad \quad \quad +3 \quad +3 \quad}$$

$$2 \sin^2 \theta - 7 \sin \theta + 3 = 0$$

$$x = \sin \theta$$

$$2x^2 - 7x + 3 = 0$$

$$(2x - 1)(x - 3) = 0$$

$$x = \frac{1}{2}, 3$$

$$\sin \theta = \frac{1}{2}$$

~~$$\sin \theta = 3$$~~



$$\theta = 30^\circ, 150^\circ$$

$$34) 2\cos^2\theta + 2\cos\theta - 1 = 0$$

$$x = \cos\theta$$

$$2x^2 + 2x - 1 = 0$$

$$a = 2$$

$$b = 2$$

$$c = -1$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4}$$

$$x = .37, -1.4$$

$$\cos\theta = .37$$

$$\cancel{68.3} \quad \cancel{68.3}$$

$$\cancel{\cos\theta = -1.4}$$

$$\hat{\theta} = \cos^{-1}.37$$

$$\theta = 68.3^\circ, 291.7^\circ$$

$$36) \sin^2 \theta - \sin \theta - 1 = 0$$

$$x = \sin \theta$$

$$x^2 - x - 1 = 0$$

$$a = 1$$

$$b = -1$$

$$c = -1$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

$$\sin \theta = \frac{1 + \sqrt{5}}{2}$$

$$\sin \theta = \frac{1 - \sqrt{5}}{2}$$

$$\theta = 321.8^\circ$$

$$\theta = 218.2^\circ$$

$$\theta = 218.2^\circ, 321.8^\circ$$

$$38) \quad \begin{array}{r} 1 - 4 \cos \theta = -2 \cos^2 \theta \\ + 2 \cos^2 \theta \qquad \qquad + 2 \cos^2 \theta \\ \hline \end{array}$$

$$2 \cos^2 \theta - 4 \cos \theta + 1 = 0$$

$$a = 2$$

$$b = -4$$

$$c = 1$$

$$X = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$$

$$X = \frac{4 \pm \sqrt{8}}{4}$$

$$\cancel{\cos \theta = 1.71}$$

$$\cos \theta = .29$$



$$\theta = 73.1^\circ, 286.9^\circ$$

$$48) \sin(2A + 50^\circ) = \frac{\sqrt{3}}{2}$$

$$\cancel{60^\circ} \quad 60^\circ \cancel{120^\circ}$$

$$\begin{array}{r} 2A + 50^\circ = 60^\circ + 360^\circ n \\ - 50^\circ \quad - 50^\circ \\ \hline \end{array}$$

$$\begin{array}{r} 2A = \frac{10^\circ}{2} + \frac{360^\circ n}{2} \\ \hline \end{array}$$

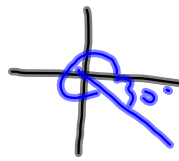
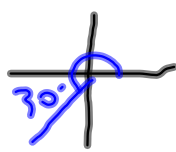
$$A = 5^\circ + 180^\circ n$$

$$\begin{array}{r} 2A + 50^\circ = 120^\circ + 360^\circ n \\ - 50^\circ \quad - 50^\circ \\ \hline \end{array}$$

$$\begin{array}{r} 2A = \frac{70^\circ}{2} + \frac{360^\circ n}{2} \\ \hline \end{array}$$

$$A = 35^\circ + 180^\circ n$$

$$52) \sin(4A - 20^\circ) = -\frac{1}{2}$$



$$4A - 20^\circ = 210^\circ + 360^\circ n$$

$$\frac{4A}{4} = \frac{230^\circ + 360^\circ n}{4}$$

$$A = 57.5^\circ + 90^\circ n$$

$$4A - 20^\circ = 330^\circ + 360^\circ n$$

$$\frac{4A}{4} = \frac{350^\circ + 360^\circ n}{4}$$

$$A = 87.5^\circ + 90^\circ n$$



