6.3- Trigonometric Equations Involving Multiple Angles

Solve $\cos 2\theta = \frac{\sqrt{3}}{2}$, if $0° \le \theta < 360°$.

$0° \le 2\theta < 720°$

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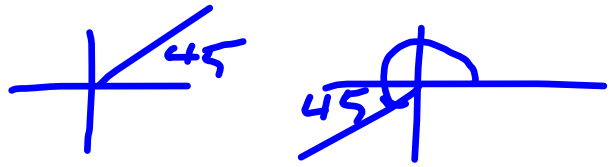
$\frac{2\theta}{2} = \frac{30°}{2}$ $\quad\quad$ $\frac{2\theta}{2} = \frac{390°}{2}$

$\frac{2\theta}{2} = \frac{330°}{2}$ $\quad\quad$ $\frac{2\theta}{2} = \frac{690°}{2}$

$\theta = 15°, 165°, 195°, 345°$

Find all solutions to $\tan 3\theta = 1$



$\frac{3\theta}{3} = \frac{45^\circ}{3} + \frac{360^\circ n}{3}$ $\qquad$ $\frac{3\theta}{3} = \frac{225^\circ}{3} + \frac{360^\circ n}{3}$

$\theta = 15^\circ + 120^\circ n \;,\; 75^\circ + 120^\circ n$

Solve: $\sin 2\theta\cos\theta+\cos 2\theta\sin\theta=\frac{\sqrt{2}}{2}$, if $0^\circ\le\theta<360^\circ$.

$\sin(2\theta+\theta)=\frac{\sqrt{2}}{2}$

$\sin 3\theta=\frac{\sqrt{2}}{2}$ $\quad 0^\circ\le 3\theta<1080^\circ$

45 135

$\frac{3\theta}{3}=\frac{45^\circ}{3}$ $\quad \frac{3\theta}{3}=\frac{405^\circ}{3}$ $\quad \frac{3\theta}{3}=\frac{765^\circ}{3}$

$\frac{3\theta}{3}=\frac{135^\circ}{3}$ $\quad \frac{3\theta}{3}=\frac{495^\circ}{3}$ $\quad \frac{3\theta}{3}=\frac{855^\circ}{3}$

$\theta=15^\circ, 45^\circ, 135^\circ, 165^\circ, 255^\circ, 285^\circ$

Homework

Problem Set 6.3: #2-44 even

Solve all problems in degrees

2) $\sin 2\theta = -\frac{\sqrt{3}}{2}$

$\frac{2\theta}{2} = \frac{240^\circ}{2}$ $\quad\quad$ $\frac{2\theta}{2} = \frac{600^\circ}{2}$

$\frac{2\theta}{2} = \frac{300^\circ}{2}$ $\quad\quad$ $\frac{2\theta}{2} = \frac{660^\circ}{2}$

$\theta = 120^\circ, 150^\circ, 300^\circ, 330^\circ$

4) $\cot 2\theta = 1$

$\tan 2\theta = 1$

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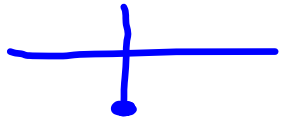
$\frac{2\theta}{2} = \frac{45^\circ}{2}$ $\frac{2\theta}{2} = \frac{405^\circ}{2}$

$\frac{2\theta}{2} = \frac{225^\circ}{2}$ $\frac{2\theta}{2} = \frac{585^\circ}{2}$

$\theta = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$

6) $\sin 3\theta = -1$



$\frac{3\theta}{3} = \frac{270^\circ}{3}$ $\frac{3\theta}{3} = \frac{630^\circ}{3}$ $\frac{3\theta}{3} = \frac{990^\circ}{3}$

$\theta = 90^\circ, 210^\circ, 330^\circ$

8) $\cos 2\theta = \frac{\sqrt{2}}{2}$

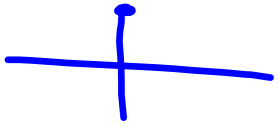
$\frac{2\theta}{2} = \frac{45}{2}^\circ$ $\quad\quad$ $\frac{2\theta}{2} = \frac{405}{2}^\circ$

$\frac{2\theta}{2} = \frac{315}{2}^\circ$ $\quad\quad$ $\frac{2\theta}{2} = \frac{675}{2}^\circ$

$\theta = 22.5^\circ, 157.5^\circ, 202.5^\circ, 337.5^\circ$

10) $\csc 3\theta = 1$

$\sin 3\theta = 1$

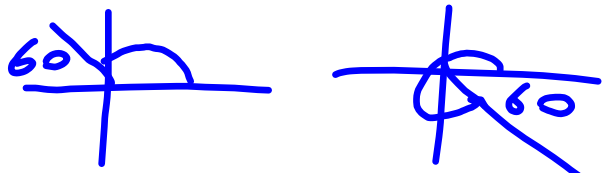


$3\theta = 90^\circ$ $\quad 3\theta = 450^\circ$ $\quad 3\theta = 810^\circ$

$3 \quad 3 \quad 3$

$\theta = 30^\circ, 150^\circ, 270^\circ$

12) $\tan 2\theta = -\sqrt{3}$



$\frac{2\theta}{2} = \frac{120^\circ}{2}$

$\frac{2\theta}{2} = \frac{480^\circ}{2}$

$\frac{2\theta}{2} = \frac{300^\circ}{2}$

$\frac{2\theta}{2} = \frac{660^\circ}{2}$

$\theta = 60^\circ, 150^\circ, 240^\circ, 330^\circ$

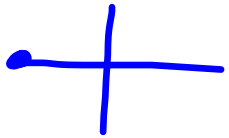
14) $\sin 2\theta = -\frac{\sqrt{3}}{2}$

60° 60°

$\frac{2\theta}{2} = \frac{240^\circ}{2} + \frac{360^\circ n}{2}$ $\frac{2\theta}{2} = \frac{300^\circ}{2} + \frac{360^\circ n}{2}$

$\theta = 120^\circ + 180^\circ n \; , \; 150^\circ + 180^\circ n$

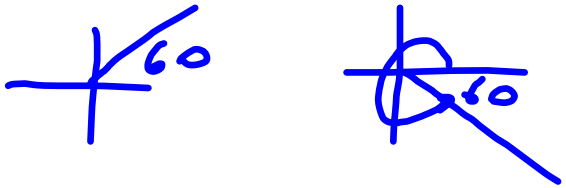
16) $\cos 3\theta = -1$



$3\theta = 180^\circ + 360^\circ n$

$\theta = 60^\circ + 120^\circ n$

18) $\cos 8\theta = \frac{1}{2}$



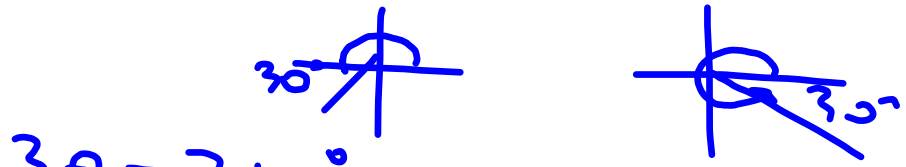
$\frac{8\theta}{8} = \frac{60^\circ}{8} + \frac{360^\circ n}{8}$ $\qquad$ $\frac{8\theta}{8} = \frac{300^\circ}{8} + \frac{360^\circ n}{8}$

$\theta = 7.5^\circ + 45^\circ n \;,\; 37.5^\circ + 45^\circ n$

20) $\sin 2\theta\cos\theta + \cos 2\theta\sin\theta = -\frac{1}{2}$

$\sin(2\theta + \theta) = -\frac{1}{2}$

$\sin 3\theta = -\frac{1}{2}$



$3\theta = 210°$ $\quad 3\theta = 570°$ $\quad 3\theta = 930°$

$3\theta = 330°$ $\quad 3\theta = 690°$ $\quad 3\theta = 1050°$

$\theta = 70°, 110°, 190°, 270°, 310°, 350°$

22) $\cos 2\theta \cos\theta - \sin 2\theta \sin\theta = \frac{\sqrt{2}}{2}$

$\cos(2\theta + \theta) = \frac{\sqrt{2}}{2}$

$\cos 3\theta = \frac{\sqrt{2}}{2}$

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$3\theta = 45^\circ$ $\quad 3\theta = 405^\circ$ $\quad 3\theta = 765^\circ$

$3\theta = 315^\circ$ $\quad 3\theta = 675^\circ$ $\quad 3\theta = 1035^\circ$

$\theta = 15^\circ, 105^\circ, 135^\circ, 225^\circ, 255^\circ, 345^\circ$

24) $\sin 2\theta \cos 3\theta + \cos 2\theta \sin 3\theta = -1$

$\sin(2\theta + 3\theta) = -1$

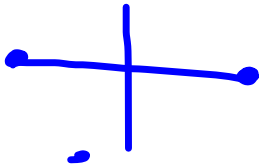
$\sin 5\theta = -1$

$5\theta = 270^\circ + 360^\circ n$

$\theta = 54^\circ + 72^\circ n$

26) $\sqrt{\cos^2 4\theta} = \sqrt{1}$

$\cos 4\theta = \pm 1$



$\frac{4\theta}{4} = \frac{0}{4} + \frac{360^\circ n}{4}$ $\qquad$ $\frac{4\theta}{4} = \frac{180^\circ}{4} + \frac{360^\circ n}{4}$

$\theta = 0 + 90^\circ n \;,\; 45^\circ + 90^\circ n$

28) $\sqrt[3]{\sin^3 5x} = \sqrt[3]{-1}$

$\sin 5x = -1$

$\frac{5x}{5} = \frac{270^\circ}{5} + \frac{360^\circ n}{5}$

$x = 54^\circ + 72^\circ n$

30) $2\sin^2 3\theta + 3\sin 3\theta + 1 = 0$

$x = \sin 3\theta$

$2x^2 + 3x + 1 = 0$

$(2x+1)(x+1) = 0$

$x = -\frac{1}{2}, -1$

$\sin 3\theta = -\frac{1}{2}$

~~30~~ ~~θ~~

$\sin 3\theta = -1$

$\frac{3\theta}{3} = \frac{210^\circ}{3} + \frac{360^\circ n}{3}$ $\quad \frac{3\theta}{3} = \frac{330^\circ}{3} + \frac{360^\circ n}{3}$

$\frac{3\theta}{3} = \frac{270^\circ}{3} + \frac{360^\circ n}{3}$

$\theta = 70^\circ + 120^\circ n, 110^\circ + 120^\circ n, 90^\circ + 120^\circ n$

32) $2\cos^2 2\theta - \cos 2\theta - 1 = 0$

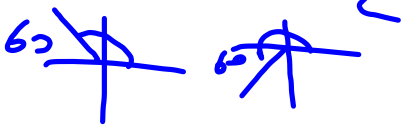
$x = \cos 2\theta$

$2x^2 - x - 1 = 0$

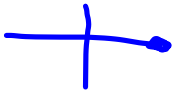
$(2x+1)(x-1) = 0$

$x = -\frac{1}{2}, 1$

$\cos 2\theta = -\frac{1}{2}$



$\cos 2\theta = 1$



$\frac{2\theta}{2} = \frac{120^\circ}{2} + \frac{360^\circ n}{2}$

$\frac{2\theta}{2} = \frac{0^\circ}{2} + \frac{360^\circ n}{2}$

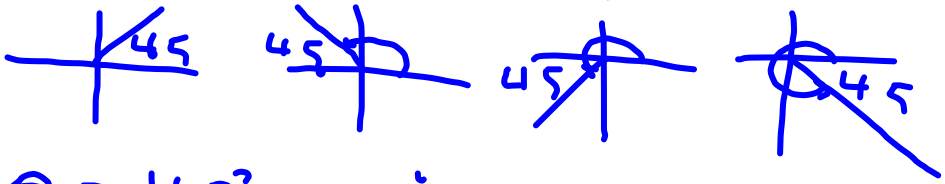
$2\theta = 240^\circ + 360^\circ n$

$\theta = 60^\circ + 180^\circ n, 120^\circ + 180^\circ n, 0 + 180^\circ n$

34) $\cot^2 3\theta = 1$

$\sqrt{\tan^2 3\theta} = \sqrt{1}$

$\tan 3\theta = \pm 1$



$3\theta = 45^\circ + 360^\circ n$ $\quad 3\theta = 135 + 360^\circ n$

$3\theta = 225^\circ + 360^\circ n$ $\quad 3\theta = 315^\circ + 360^\circ n$

$\theta = 15^\circ + 120^\circ n,\ 45^\circ + 120^\circ n,$

$75^\circ + 120^\circ n,\ 105^\circ + 120^\circ n$

36) $(\sin\theta - \cos\theta)^2 = 1^2$

$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 1$

$1 - 2\sin\theta\cos\theta = 1$

$-1 \qquad\qquad -1$

$-2\sin\theta\cos\theta = 0$

-1

$-\sin 2\theta = 0$

$\sin 2\theta = 0$

$\frac{2\theta}{2} = \frac{0^\circ}{2}$ $\qquad$ $\frac{2\theta}{2} = \frac{360^\circ}{2}$

$\frac{2\theta}{2} = \frac{180^\circ}{2}$ $\qquad$ $\frac{2\theta}{2} = \frac{540^\circ}{2}$

$\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$

38) $(\cos\theta - \sin\theta)^2 = (-1)^2$

$\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 1$

$1 - 2\sin\theta\cos\theta = 1$

$-1 \quad\quad -1$

$-2\sin\theta\cos\theta = 0$

$-1\left[-\sin 2\theta = 0\right]$

$\sin 2\theta = 0$

$\frac{2\theta}{2} = \frac{0}{2}$ $\quad\quad$ $\frac{2\theta}{2} = \frac{360^\circ}{2}$

$\frac{2\theta}{2} = \frac{180^\circ}{2}$ $\quad\quad$ $\frac{2\theta}{2} = \frac{540^\circ}{2}$

$\theta = \cancel{0^\circ}, 90^\circ, 180^\circ, \cancel{270^\circ}$

40) $\cos^2 3\theta - 6\cos 3\theta + 4 = 0$

$x = \cos 3\theta$

$x^2 - 6x + 4 = 0$

$a = 1$, $b = -6$, $c = 4$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$x = 5.23, .76$

~~$\cos 3\theta = 5.23$~~

$\cos 3\theta = .76$

$\frac{3\theta}{3} = \frac{40.5^\circ}{3}$ $\quad \frac{3\theta}{3} = \frac{400.5}{3}$ $\quad \frac{3\theta}{3} = \frac{760.5}{3}$

$\frac{3\theta}{3} = \frac{319.5^\circ}{3}$ $\quad \frac{3\theta}{3} = \frac{679.5}{3}$ $\quad \frac{3\theta}{3} = \frac{1039.5^\circ}{3}$

$\theta = 13.5^\circ, 106.5^\circ, 133.5^\circ, 226.5^\circ, 253.5^\circ, 346.5^\circ$

42) $2\sin^2 2\theta - 6\sin 2\theta + 3 = 0$

$x = \sin 2\theta$

$2x^2 - 6x + 3 = 0$

$a = 2$
$b = -6$
$c = 3$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{4}$$

$$= \frac{6 \pm \sqrt{12}}{4}$$

$x = 2.37, .63$

~~$\sin 2\theta = 2.37$~~

$\sin 2\theta = .63$

39 ~~37~~

$\frac{2\theta}{2} = \frac{39}{2}$

$\frac{2\theta}{2} = \frac{399}{2}$

$2\theta = 141^\circ$

$2\theta = 501^\circ$

$\theta = 19.5^\circ, 70.5^\circ, 199.5^\circ, 250.5^\circ$

44) $2\sin^2 4\theta - 2\cos 4\theta = 1$

$2(1-\cos^2 4\theta) - 2\cos 4\theta = 1$

$2 - 2\cos^2 4\theta - 2\cos 4\theta = 1$

$-1 \quad\quad -1$

$-1[-2\cos^2 4\theta - 2\cos 4\theta + 1 = 0]$

$2\cos^2 4\theta + 2\cos 4\theta - 1 = 0$

$a=2$, $b=2$, $c=-1$

$$\theta = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4+8}}{4}$$

$$= \frac{-2 \pm \sqrt{12}}{4}$$

$\theta = .37, -1.37$

$\cos 4\theta = .37$ ~~$\cos 4\theta = 1.37$~~

63.3, 68.3

$\frac{4\theta}{4} = \frac{68.3^\circ}{4}$ $\frac{4\theta}{4} = \frac{428.3^\circ}{4}$ $\frac{4\theta}{4} = \frac{788.3^\circ}{4}$ $\frac{4\theta}{4} = \frac{1148.3^\circ}{4}$

$\frac{4\theta}{4} = \frac{291.7^\circ}{4}$ $\frac{4\theta}{4} = \frac{651.7^\circ}{4}$ $\frac{4\theta}{4} = \frac{1011.7}{4}$ $\frac{4\theta}{4} = \frac{1371.7^\circ}{4}$

$\theta = 17.1^\circ, 72.9^\circ, 107.1^\circ, 162.9^\circ, 197.1^\circ, 252.9^\circ, 287.1^\circ, 342.9^\circ$