P.1 Graphs and Models

Graphing Linear Equations

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope-Intercept Form: $y = mx + b$

-- solve the equation for y

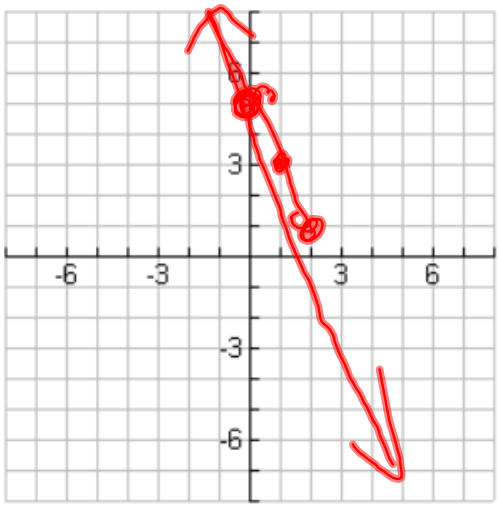
-- m = slope

= rise over run

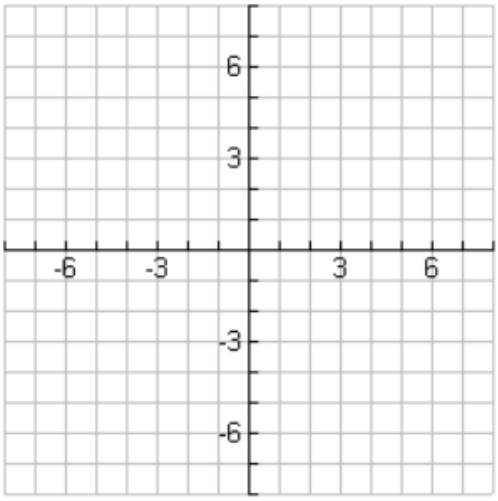
$= \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

-- b = y-intercept

ex 1) $2x + y = 5$



2) $3y - 4x = -9$



Graphs of higher order equations:

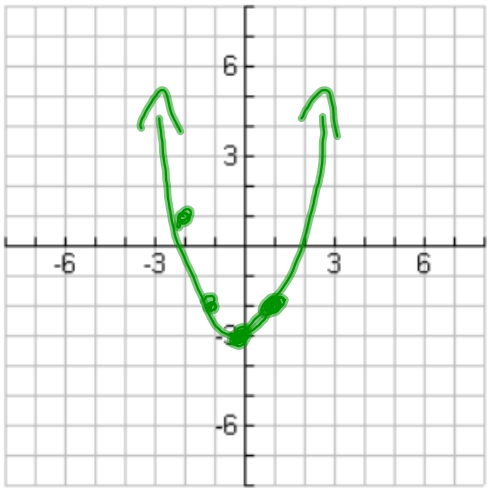
-- use a table of values to plot points.

ex 3) $y + 3 = x^2$

$y = x^2 - 3$ solve for y.

x	y

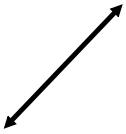
make table of values.
(choose easiest points)



Graph a complete & smooth curve

Basic curves:

Linear
x



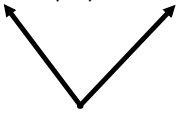
Quadratic
x^2

Cubic
x^3

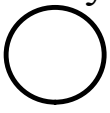
Quartic
x^4

Square Root
$\sqrt{x}$

Absolute Value
$|x|$



Circle
$x^2 + y^2$



Intercepts of a Graph

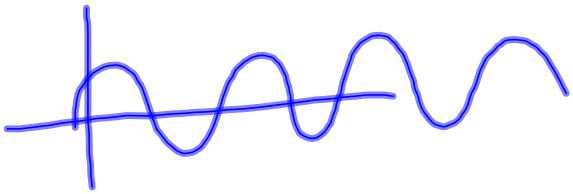
x-intercept -- where the graph intersects the x-axis
-- point (a, 0)
-- y-value always = 0

ex) $y = x^2 - 16$ Let y = 0 and solve for x

$0 = x^2 - 16$

$x = \pm 4$ (4,0) (-4,0)

* How many x-int can a graph have?

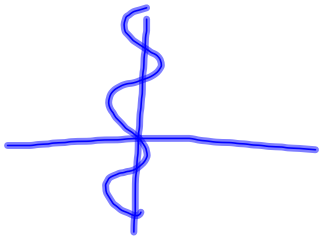


y-intercept -- where the graph intersects the y-axis
-- point (0, b)
-- x-value always = 0

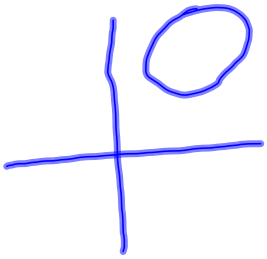
ex) $y = x^2 - 16$ Let x=0 and solve for y

$y = 0 - 16$ (0,-16)

* How many y-intercepts can a graph have?



* Does a graph have to have any intercepts?



ex: Find all intercepts for:

$x^2 + 3x = y^2 + 4$

$x^2 + 3x = 0 + 4$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$(-4, 0)(1, 0)$

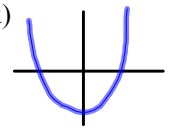
$0 = y^2 + 4$

$\sqrt{y^2} = \sqrt{-4}$

Symmetry

3 Basic Symmetries:

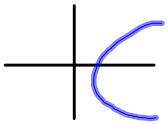
1) y-axis sym -- reflect across y-axis
   -- if point (x,y) is on the graph, then (-x,y) is also on the graph
   -- replacing x with -x results in the same eqn.
   -- Even function

ex) 

$y = x^2 - 5$

$y = (-x)^2 - 5 = x^2 - 5$

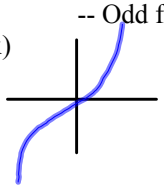
2) x-axis sym -- reflect across x-axis
   -- if point (x,y) is on the graph, then (x,-y) is also on the graph
   -- replacing y with -y results in the same eqn.
   -- not a function

ex) 

$x = y^2 + 2$

$x = (-y)^2 + 2 = y^2 + 2$

3) origin sym -- reflect through the origin (0,0)
   -- if point (x,y) is on the graph, then (-x,-y) is also on the graph
   -- replacing x with -x, and y with -y results in the same equation.
   -- Odd function

ex) 

$y = x^3$

$-y = (-x)^3$

$-y = -x^3$

$y = x^3$

ex1: $x^2 - 3y^2 = 7$

y-axis

$(-x)^2 - 3y^2 = 7$

$x^2 - 3y^2 = 7$

x-axis

$x^2 - 3(-y)^2 = 7$

$x^2 - 3y^2 = 7$

origin

$(-x)^2 - 3(-y)^2 = 7$

$x^2 - 3y^2 = 7$

ex2) $x = 5y^3$

$y = 5x^3$

Points of Intersection

2 Methods:

1) solve each eqn for the same variable, then set them equal to each other.

2) solve one eqn for a single variable, then substitute into the other eqn.

ex 1) $x^2 - y = 3$ and $x - y = 1$ $\rightarrow x = 1 + y$

$(1+y)^2 - y = 3$

$y^2 + 2y + 1 - y = 3$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2, 1$

$(-1, -2)$
$(2, 1)$

ex 2) $x^2 + y^2 = 5$ and $x - y = 1$

$(-1, -2)$
$(2, 1)$

* How many intersection points can 2 graphs have?

$9 = x^2 \sqrt{25 - x^2}$

$x^2 = 0 \quad \sqrt{25 - x^2} = 0$

HW: p 8, # 1-4, 9, 12, 17, 19, 22, 23, 28-36e, 60-70e, 76