

3.1 Extrema on an Interval

Absolute Min/Max

1.  $f(c)$  is the **minimum** of  $f$  on an interval if  $f(c) \leq f(x)$  for all  $x$  in the interval.
2.  $f(c)$  is the **maximum** of  $f$  on an interval if  $f(c) \geq f(x)$  for all  $x$  in the interval.

Extreme Value Theorem

If  $f$  is continuous on a closed interval  $[a,b]$ , then  $f$  has both a min & max on the interval.

Relative Extrema

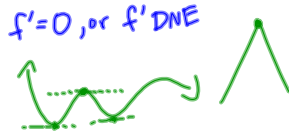
1. If there is an open interval containing  $c$  on which  $f(c)$  is a max, then  $f(c)$  is a **relative maximum**.
2. If there is an open interval containing  $c$  on which  $f(c)$  is a min, then  $f(c)$  is a **relative minimum**.



Critical Number

Let  $f$  be defined at  $c$ . If  $f'(c) = 0$  or if  $f$  is not differentiable at  $c$ , then  $c$  is a critical number of  $f$ .

\*\*Relative Extrema occur only at critical numbers



Oct 10-3:20 PM

To find extrema on  $[a,b]$ :

1. find all critical numbers in  $(a,b)$   $f'=0, DNE$
2. Evaluate  $f$  at each critical number
3. Evaluate  $f$  at each endpoint of  $[a,b]$
4. Locate min & max value

ex1) Find extrema of  $f(x) = 3x^4 - 4x^3$  on  $[-2,2]$

$$f' = 12x^3 - 12x^2 = 0$$

$$12x^2(x-1) = 0$$

$$x = 0, 1$$

$$f(0) = 0$$

$$f(1) = -1$$

$$f(-2) = 80$$

$$f(2) = 16$$

$$\text{max } (-2, 80)$$

$$\text{min } (1, -1)$$

Oct 10-3:32 PM

ex2) Find extrema of  $f(x) = 2x - 3x^{2/3}$  on  $[-1,3]$

$$f' = 2 - 2x^{-1/3} = 0$$

DNE @  $x=0$

$$2 - \frac{2}{\sqrt[3]{x}} = 0$$

$$2 = \frac{2}{\sqrt[3]{x}}$$

$$2\sqrt[3]{x} = 2$$

$$\sqrt[3]{x} = 1$$

$$x = 1$$

$$f(0) = 0 \rightarrow \text{max } (0, 0)$$

$$f(1) = -1$$

$$f(-1) = -5 \rightarrow \text{min } (-1, -5)$$

$$f(3) = -240$$

Oct 10-3:35 PM

ex3) Find extrema of  $f(x) = 2 \sin x - \cos(2x)$  on  $[0, 2\pi]$

$$f' = 2\cos x + 2\sin(2x)$$

$$2\cos x + 2\sin(2x) = 0$$

$$2\cos x + 2(2\sin x \cos x) = 0$$

$$\cos x + 2\sin x \cos x = 0$$

$$\cos x(1 + 2\sin x) = 0$$

$$\cos x = 0$$

$$x = \pi/2, 3\pi/2$$

$$\sin x = -1/2$$

$$x = 7\pi/6, 11\pi/6$$

$$f(\pi/6) = 2\sin(\pi/6) - \cos(\pi/3) = 3$$

$$f(5\pi/6) = 2\sin(5\pi/6) - \cos(5\pi/3) = -1$$

$$f(7\pi/6) = 2\sin(7\pi/6) - \cos(7\pi/3) = -3/2$$

$$f(11\pi/6) = -3/2$$

$$f(0) = -1$$

$$f(2\pi) = -1$$

$$\text{min } (-3/2) \text{ at } (7\pi/6, -3/2) \text{ and } (11\pi/6, -3/2)$$

$$\text{max } (3) \text{ at } (\pi/6, 3)$$

Oct 10-3:37 PM

hw p165 #1-10, 17-31 odd

Oct 10-3:40 PM