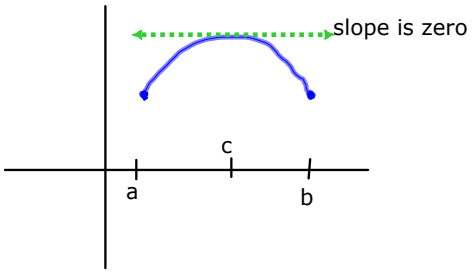
3.2 Rolle's Theorem & Mean Value Theorem

Rolle's Thm

Let f(x) be continuous on [a,b] and differentiable on (a,b).

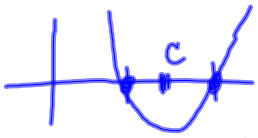
If f(a) = f(b),
then there is at least one number 'c' in the interval such that f'(c) = 0



Nov 8-9:11 AM

ex1) Find the 2 x-intercepts of
$f(x) = x^2 - 3x + 2$
and show that f'(c) = 0 for some point between the intercepts.

x-int: x = 1, 2

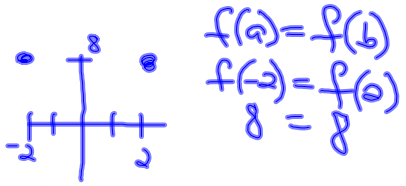


$f' = 2x - 3 = 0$

$x = 3/2$

Nov 8-9:19 AM

ex2) $f(x) = x^4 - 2x^2$
Find all numbers on the interval (-2,2) such that f'(c) = 0



$f(a) = f(b)$

$f(-2) = f(0)$

$8 = 8$

$f' = 4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$x = 0, \pm 1$

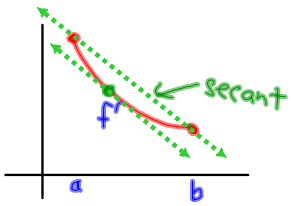
Nov 8-9:24 AM

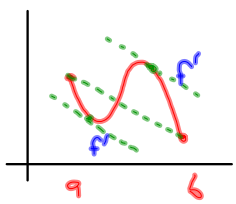
Mean Value Theorem (MVT) for Derivatives

If f(x) is continuous on [a,b] and differentiable on (a,b),

then there exists some number 'c' such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$





Nov 8-9:29 AM

ex) f(x) = 5 - 4/x

Find all numbers on [1,4] guaranteed by MVT

$5-4x^{-1}$

Discont @ $x=0$

cont on $[1,4]$

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$\frac{4}{x^2}=\frac{f(4)-f(1)}{4-1}$$

$$\frac{4}{x^2}=\frac{4-1}{3}\rightarrow\frac{4}{x^2}=1$$

$$x^2=4$$

$$x=\pm 2$$

$$x=2$$

Nov 8-9:35 AM

ex) Show that the MVT can be applied on $[0, \pi]$, then find all 'c' values it guarantees for $f(x) = 2\sin x + \sin(2x)$

$$f'=\frac{f(\pi)-f(0)}{\pi-0}$$

$$2\cos x+2\cos(2x)=\frac{0-0}{\pi}$$

$$2\cos x+2\cos(2x)=0$$

$$\cos x+\cos(2x)=0$$

$$\cos x+2\cos^2 x-1=0$$

$$2\cos^2 x+\cos x-1=0$$

$$(2\cos x-1)(\cos x+1)$$

$\cos x = 1/2$ $\quad$ $\cos x = -1$

$x = \pi/3$ $\quad$ $x = \pi$

Nov 8-9:40 AM

p172 #1-3, 7, 13, 17, 26, 33, 35, 37, 43

Nov 8-9:46 AM