

Limits at Infinity (3.5)
 (y-values)
 → a function value as x approaches ∞ or $-\infty$
 → these are horizontal Asymptotes

Guidelines:
 Look at the degree of the Numerator & Denominator.

- 1) If deg. of top > deg. of bottom there is NO H.A.
- 2) If deg. of top = deg. of bottom the H.A. = leading coefficient of the top divided by lead. coeff. of bottom.
 ex) $f(x) = \frac{3x^2 + 7x + 1}{2x^2 + 5}$
 H.A. $y = \frac{3}{2}$
- 3) If deg. top < deg. bottom, then the H.A. is $y = 0$
 ex) $f(x) = \frac{3x^3 - 4x + 2}{x^4 + 1}$

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ex) Find H.A. for
 $f(x) = \frac{2x^3 + 3x^2 + 1}{10x^3 - 3x}$

Divide all terms by the highest power of Denom. → x^3

$$\lim_{x \rightarrow \infty} \frac{\frac{2x^3}{x^3} + \frac{3x^2}{x^3} + \frac{1}{x^3}}{\frac{10x^3}{x^3} - \frac{3x}{x^3}}$$

$$\lim_{x \rightarrow \infty} \frac{2 + \frac{3}{x} + \frac{1}{x^3}}{10 - \frac{3}{x^2}} = \frac{2 + 0 + 0}{10 - 0} = \frac{2}{10} = \frac{1}{5}$$

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2) Find all H.A. for
 $f(x) = \frac{3x - 2}{\sqrt{2x^2 + 1}}$

$$\lim_{x \rightarrow \infty} f(x) = \frac{\frac{3x}{x} - \frac{2}{x}}{\frac{\sqrt{2x^2 + 1}}{\sqrt{x^2}}} = \frac{3 - \frac{2}{x}}{\sqrt{2 + \frac{1}{x^2}}} = \frac{3}{\sqrt{2}}$$

$$\lim_{x \rightarrow -\infty} f(x) = \frac{\frac{3x}{x} - \frac{2}{x}}{\frac{\sqrt{2x^2 + 1}}{-\sqrt{x^2}}} = \frac{3 - \frac{2}{x}}{-\sqrt{2 + \frac{1}{x^2}}} = -\frac{3}{\sqrt{2}}$$

Find all HA for
 $f = \frac{\sqrt{5x^2 + 3}}{4 - 2x}$

$$\lim_{x \rightarrow \infty} f = \frac{\sqrt{5}}{2}$$

$$\lim_{x \rightarrow -\infty} f = \frac{\sqrt{5}}{2}$$

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3) Find HA's for
 $g(x) = \frac{\sqrt{5x^4 + 2x^2 + 1}}{x^2 - x + 2}$

$$\lim_{x \rightarrow \infty} g(x) = \sqrt{5}$$

$$\lim_{x \rightarrow -\infty} g(x) = -\sqrt{5}$$

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P197 Trig Limits

1) $\lim_{x \rightarrow \pm\infty} \cos x = \text{DNE}$

2) $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$

$$\frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$
$$0 \leq \lim_{x \rightarrow \pm\infty} \frac{\sin x}{x} \leq 0$$

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1-6, 13-31 odd

Nov 5-11:46 AM