3.7 Optimization Problems

-- Finding the "best" solution to a problem

-- Locate the max or min of an eqn.

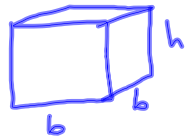
-- where f '(x) = 0 , or f '(x) DNE

Primary Equation
    -- the eqn to be optimized

Secondary Equation
    -- any eqn(s) needed for substitution into
        the primary eqn

Oct 30-4:49 PM

ex1) an open top box is to be made with a square base and a surface area of 108 in^2. What dimensions will give largest volume?



$V = b^2 h$

$V = b^2\left(\frac{108-b^2}{4b}\right)$

$V = \frac{108b - b^3}{4}$

$V = \frac{1}{4}\left(108b - b^3\right)$

$\frac{dV}{db} = \frac{1}{4}\left(108 - 3b^2\right)$

$0 = \frac{1}{4}\left(108 - 3b^2\right)$

$3b^2 = 108$

$b^2 = 36$

$b = \pm 6$

$SA = b^2 + 4bh$

$108 = b^2 + 4bh$

$108 - b^2 = 4bh$

$\frac{108-b^2}{4b} = h$

$V'' = \frac{1}{4}(-6b)$

$V''(6) = \text{Neg}$

cc↓, so max

$\boxed{b = 6 \text{ in}}$ $\boxed{h = 3 \text{ in}}$

Nov 29-9:23 AM

2) Find the points on the graph of $y = 4 - x^2$ that are closest to (0, 2)

$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$D = \sqrt{(x-0)^2 + (y-2)^2}$

$D = \sqrt{x^2 + (4 - x^2 - 2)^2}$

$D^2 = x^2 + (2 - x^2)^2$

$= x^2 + 4 - 4x^2 + x^4$

$f = x^4 - 3x^2 + 4$

$f' = 4x^3 - 6x = 0$

$2x(2x^2 - 3) = 0$

$x = 0, \pm\sqrt{3/2}$

$f'' = 12x^2 - 6$

$f''\left(\pm\sqrt{3/2}\right) = +$ cc↑ min.

$f''(0) = -$ cc dn ∩

$y = 4 - x^2$

$\left(\pm\sqrt{3/2}, 5/2\right)$

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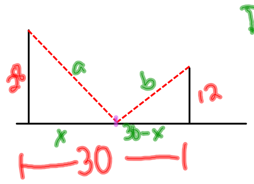
3) Two posts are 30 feet apart. One is 12 feet high, & the other is 28 feet high.
They are connected to the ground with wires running from the top of each post to a stake located on the ground between them.
Where should the stake be located to use the smallest amount of wire?



$D = a + b$

$28^2 + x^2 = a^2$

$a = \sqrt{28^2 + x^2}$

$b = \sqrt{12^2 + (30-x)^2}$

$D = \sqrt{28^2 + x^2} + \sqrt{12^2 + (30-x)^2}$

$= \sqrt{28^2 + x^2} + \sqrt{1044 - 60x + x^2}$

$= \frac{1}{2}(28^2 + x^2)^{-1/2}(2x) + \frac{1}{2}(1044 - 60x + x^2)^{-1/2}(2x - 60)$

$\frac{x}{\sqrt{28^2 + x^2}} + \frac{x - 30}{\sqrt{1044 - 60x + x^2}} = 0$

$\left(\frac{x}{\sqrt{28^2 + x^2}}\right)^2 = \left(\frac{-x + 30}{\sqrt{1044 - 60x + x^2}}\right)^2$

$x^2(1044 - 60x + x^2) = (30 - x)^2(28^2 + x^2)$

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p208 #1-6, 55, 57, 66
p 216 #7, 9, 11, 15-17, 23, 24, 27, 30, 34, 40

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