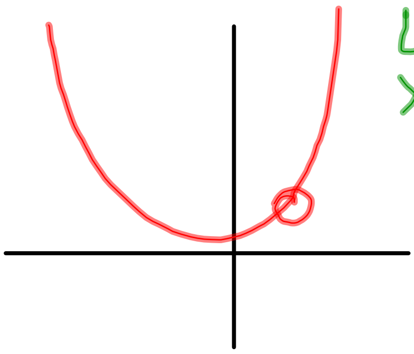
1.2 Finding Limits Graphically and Numerically

Limit -- the y-value that a function approaches as the x-value approaches a given constant.

$\lim_{x \to c} f(x) = L$ "the limit of f(x) as x approaches c is L"

ex1) $f(x) = \dfrac{x^3 - 1}{x - 1}$ Find $\lim_{x \to 1} f(x) =$

Use x-values that get closer to 1 from the right and from the left.



$\lim_{x \to 1} f(x) = 3$

x	.5	.75	.9	.99	1	1.01	1.1	1.25	1.5
f(x)					?				

ex2) $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x - 2}$

ex3) $\lim_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1}$

$\lim_{x \to 0} f(x) = 2$

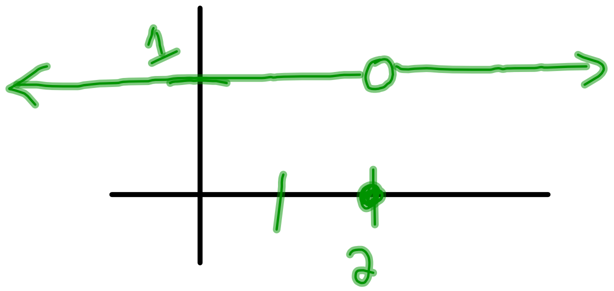
Existence of Limits

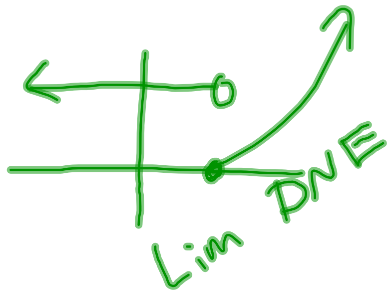
-- in order for a limit to exist, it must approach the same value from the right and the left.

ex4) Find the limit of f(x) as x approaches 2

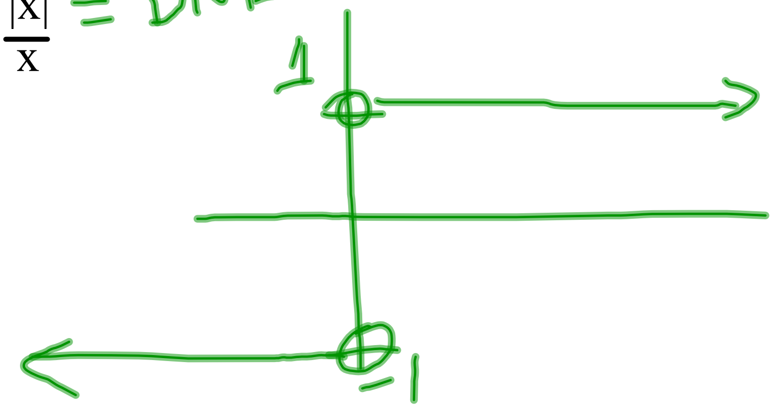
$$f(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

$\lim_{x \to 2} f(x) = 1$





5) $\lim_{x \to 0} \frac{|x|}{x}$ = DNE



Unbounded Behavior → No Limit, approaches $\pm\infty$

6) $\lim_{x \to 0} \frac{1}{x^2}$

DNE

approach ∞

Oscillating Behavior of a Graph

ex) $\lim_{x \to 0} \sin(1/x)$ = DNE

HW: p.54 # 1-7 odd, 8-18, 44, 45

HW2: p 55 # 23-37 odd, 49-52

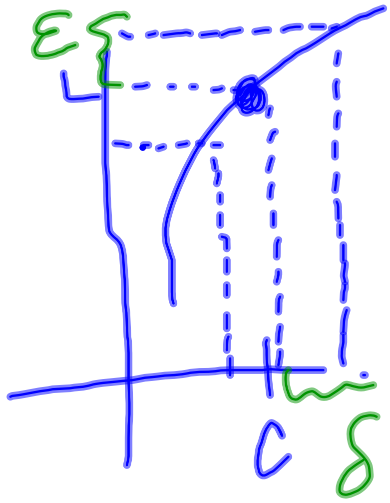
Definition of a Limit

If f(x) is a function on an open interval containing 'c',

Then $\lim\limits_{x \to c} f(x) = L$

Means that for $\epsilon > 0$
there exists $\delta > 0$ such that:

If $0 < |x-c| < \delta$, then $|f(x) - L| < \epsilon$



ex1) $\lim_{x \to 3} (2x-5) = 1$

Find the limit and prove that is exists using the ξ-δ definition of Limits.

$0 < |x-c| < \delta \quad , \quad |f(x)-L| < \varepsilon$

$0 < |x-3| < \delta \quad , \quad |(2x-5) - 1| < \varepsilon$

$|2x-6| < \varepsilon$

$2|x-3| < \varepsilon$

$|x-3| < \varepsilon/2$

$\boxed{\delta = \varepsilon/2}$

ex2) $\lim_{x \to -2} (3x-4) = -10$

Find the limit and find δ such that
$|(3x-4)-L| < .01$ when $0 < |x+2| < \delta$

$f(x)$ ε $x-c$

$|3x-4+10| < .01$, $|x+2| < \delta$

$|3x+6| < .01$

$3|x+2| < .01$

$\delta = \frac{.01}{3}$

$= 1/300$

ex1) $\lim_{x \to 2} x^2 = 4$

Find the limit and prove that is exists using the ξ-δ definition of Limits.

$0 < |x-2| < \delta$, $|x^2-4| < \varepsilon$

$1 < x < 3$

$|(x+2)(x-2)| < \varepsilon$

$|x-2| < \frac{\varepsilon}{x+2}$

$\delta = \varepsilon/5$

$|x-2| < \varepsilon/5$

2) $f(x) = \begin{cases} 2x+1, & x < 3 \\ 5-x, & x \ge 3 \end{cases}$

$\lim_{x \to 3} f(x)$

$2(3)+1 \overset{?}{=} 5-(3)$

$7 \neq 2$

DNE