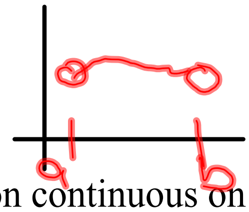
1.4 Continuity and One-Sided Limits

Continuity

A function is *continuous* at *c* if 3 conditions are met:

1. *f(c)* is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

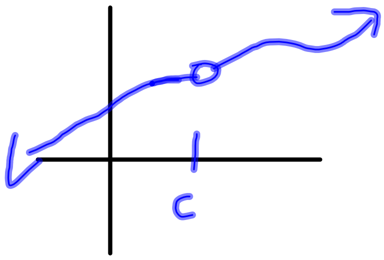
A function is continuous on an *open interval* (a, b) if it is continuous at each point in the interval.



A function continuous on the entire real line is called *everywhere continuous*.

Discontinuities:

1. the function has a hole in the graph at a point x = c.



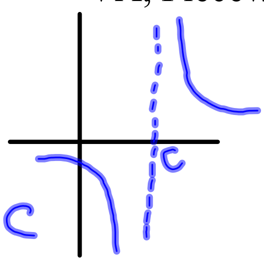
$f(x) = \frac{(x-c)(x+5)}{x-c}$

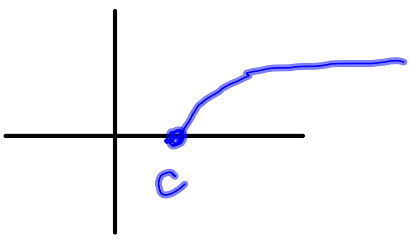
Hole @ c

2. the limit of f(x) does not exist at x = c.
   VA, Piecewise, or Square Root

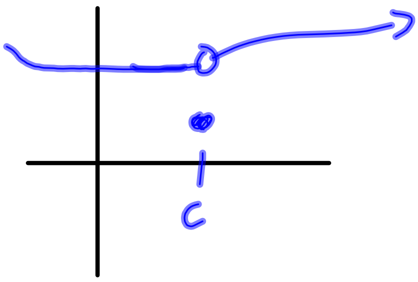
$\frac{x}{x-c}$

V.A@c





3. the limit of f(x) exists, but is not equal to f(c).



Piecewise

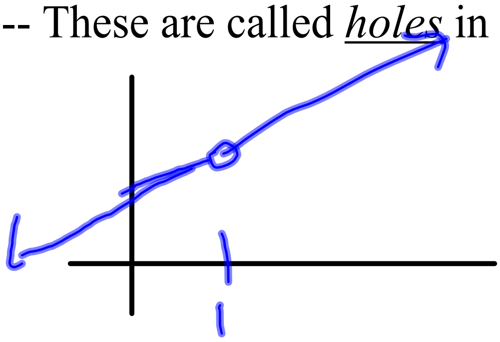
2 Types of Discontinuities:

1. <u>Removable Discont.</u> -- the function can be reduced or redefined as a continuous function.

   ex: factor and cancel out the discont. point.

   $$f(x) = \frac{x^2 + 7x - 8}{x - 1} = \frac{\cancel{(x-1)}(x+8)}{\cancel{x-1}} = x+8$$

   -- These are called *holes* in the graph.



2. <u>Nonremovable Discont.</u> -- the function cannot be redefined as a continuous function.

   ex: vertical asymptotes & gaps

   $$f(x) = \frac{x^2 - 16}{x + 3}$$ VA @ $x = -3$

Find all discont. & tell if each is removable or non-rem.
Find the intervals on which each function is continuous:

1. $f(x) = 1 / x$

Discont: $x=0$ Non-Remov. VA

Cont: $(-\infty,0)(0,\infty)$

2. $g(x) = \dfrac{x^2 - 1}{x - 1}$

Discont: $x=1$ Remov. (Hole)

Cont: $(-\infty,1)(1,\infty)$

3. $h(x) = \begin{cases} x + 1, & x \le 0 \\ x^2 + 1, & x > 0 \end{cases}$

$(0,1)$
$(0,1)$

everywhere cont.

$(-\infty,\infty)$

4. $y = (\sin x)/(2x)$

$\frac{1}{2}\cdot\frac{\sin x}{x}$

Discont @ $x=0$ Non Rem

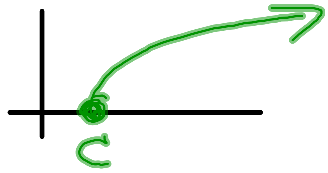
cont $(-\infty,0)(0,\infty)$

One-Sided Limits

A limit can be looked at from either direction:

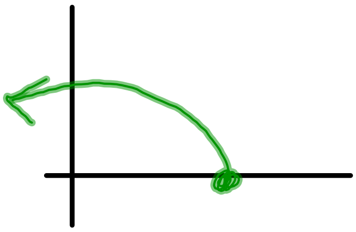
Limit from the right -- x approaches c from values larger than c.

$$\lim_{x \to c^+} f(x) = L$$

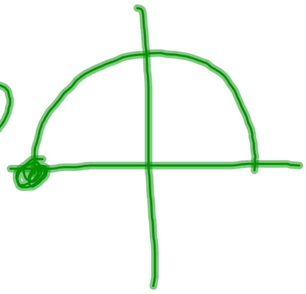


Limit from the left -- x approaches c from values smaller than c

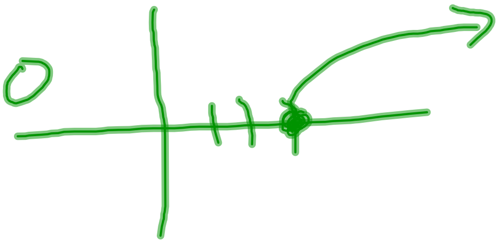
$$\lim_{x \to c^-} f(x) = L$$



ex1) $\lim_{x \to -2^+} \sqrt{4 - x^2} = 0$



ex2) $\lim_{x \to 3^+} \sqrt{x - 3} = 0$



ex3) $\lim_{x \to 3^-} \sqrt{x - 3}$ = DNE

Existence of a Limit

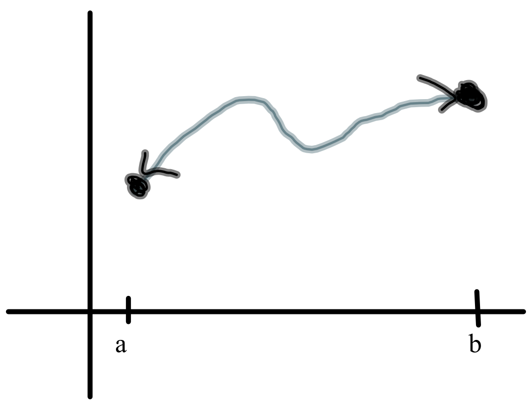
The limt of f(x) as x approaches c is a real number L if and only if:

$$\lim_{x \to c^-} f(x) = L \quad \text{and} \quad \lim_{x \to c^+} f(x) = L$$

Continuity on a closed interval

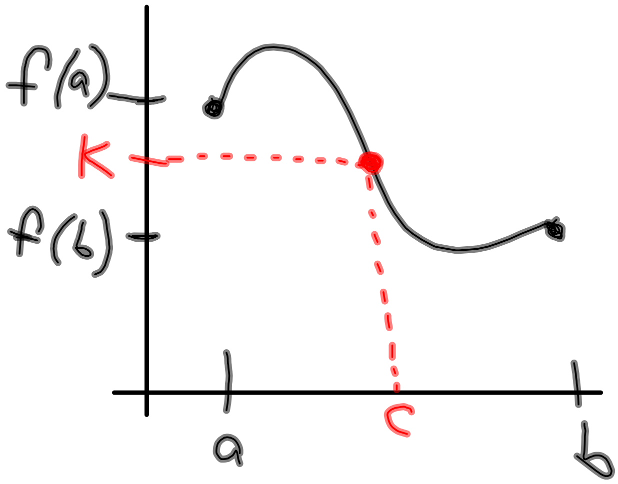
A function is continuous on the closed interval [a, b] if it is continuous on the open interval (a, b) and if:

$$\lim_{x \to a^+} f(x) = f(a) \quad \& \quad \lim_{x \to b^-} f(x) = f(b)$$



Intermediate Value Theorem

If f is continuous on the closed interval [a, b] and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in [a, b] such that $f(c) = k$.



ex1) Use the Intermediate Value Thm. to show that the polynomial $f(x) = x^3 + 2x - 1$ has a zero in the interval [0, 1].

cont on [0, 1]

$f(0) = -1$

$f(1) = 1 + 2 - 1 = 2$

It must cross y=0 at some c in [0,1]

ex2) Verify that the Intermediate Value Thm applies on [0, 4] and find c such that f(c)=2 for the function $f(x)=x^2+2x-6$

cont everywhere

$f(0) = -6$

$f(4) = 18$

$x^2+2x-6=2$

$x^2+2x-8=0$

$x = -4, 2$

ex3) Verify that the Intermediate Value Thm applies on [-1, 10] and find c such that f(c)=2/3 for f(x)=(x+3)/(x+5)

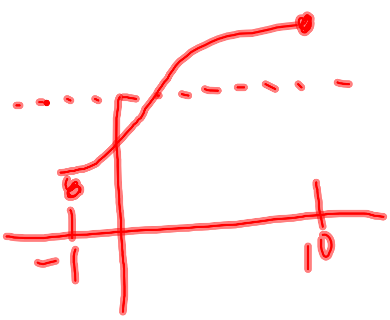
Discont $x=-5$

cont on $[-1,10]$

$f(-1) = 1/2$

$f(10) = 13/15$

$$\frac{x+3}{x+5} = \frac{2}{3}$$



$x = 1$

HW: p 76 #3,6, 9-12, 15-21, 27, 35-51odd, 59
77, 85

HW2: p 77 #34, 38, 42, 48, 58, 78, 84, 86, 88
turn in