2.1 Derivatives

Symbols: f '(x), dy/dx, d/dx (f(x)), y' , $D_x[y]$

Derivative = Slope

Definition of Derivative:

$$f'(x) = \lim_{h \Rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\boxed{f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

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ex1) $f(x) = x^2 - 5x$
Find f '(x), and find the slope of the curve at x=-2

$$\lim_{h \to 0} \frac{(x+h)^2 - 5(x+h) - (x^2 - 5x)}{h}$$

$$\frac{\cancel{x^2} + 2xh + h^2 - \cancel{5x} - 5h - \cancel{x^2} + \cancel{5x}}{h}$$

$$\lim_{h \to 0} (2x + h - 5)$$

$$f'(x) = 2x - 5$$

$$\boxed{f'(-2) = -9}$$

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ex2a) f(t) = 2/t
Find f '(t), and find the slope of the curve when t=4

$$f'(4) = -\frac{1}{8}$$

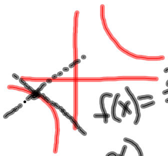
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\frac{t}{t}\cdot\frac{2}{t+h} - \frac{2}{t}\cdot\frac{t+h}{t+h}}{h}$$

$$\frac{\cancel{2t} - \cancel{2t} - 2h}{t(t+h)} \cdot \frac{1}{h} = \frac{-2}{t^2}$$

2b) Write the equation of the tangent and normal lines at the point where x=-3

$$f'(t) = \frac{-2}{t^2}$$



$f(x) = \frac{2}{t}$

$$f'(-3) = -2/9$$

$(-3, -2/3)$

tangent

$$y - y_1 = m(x - x_1)$$

$$\boxed{y + 2/3 = \frac{-2}{9}(x+3)}$$

Normal ⊥
$m = 9/2$

$$\boxed{y + 2/3 = \frac{9}{2}(x+3)}$$

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ex3) Find the equation for the line tangent to $f(x) = \sqrt{x}$, when x = 9

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \frac{\cancel{x} + \cancel{h}^{1} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$f'(9) = \frac{1}{6}$ $\quad f(9) = \sqrt{9} = 3$

$$3 = \frac{1}{6}(9) + b$$

$$3/2 = b$$

$$\boxed{y = 1/6\, x + 3/2}$$

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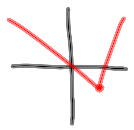
Existence of the Derivative

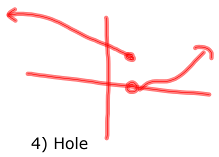
- the function must be continuous at the given point.
- the limit must exist from both sides, and $\lim_{x \to c^+} = \lim_{x \to c^-}$
- must be a smooth curve at the given point. (no sharp corners)

The Deriv. does NOT exist at:

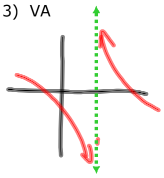
1) Sharp Turn



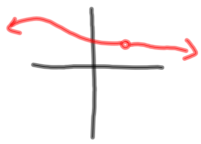
2) Break/gap



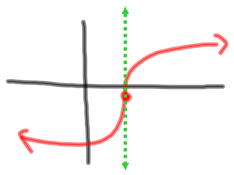
3) VA



4) Hole



5) Vertical Tangent (slope undef.)



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ex) Find an eqn. tangent to $f(x) = x^3$ and parallel to $3x-y+1=0$ $\rightarrow y=3x+1$

$m=3$

$$f' = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$\frac{\cancel{x^3} + h^3 + 3h^2x + 3hx^2 - \cancel{x^3}}{h}$$

$$\lim_{h \to 0} h^2 + 3hx + 3x^2$$

$$f' = 3x^2 = 3$$

$$x = \pm 1$$

$f = x^3$

$(1,1)$ | $(-1,-1)$

$y-1 = 3(x-1)$ | $y+1 = 3(x+1)$

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Thm 2.1 Differentiability Implies Continuity

If $f(x)$ is differentiable at x=c, then f is continuous at x=c.

Alternate Form of Derivative:
Derivative at a point

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

ex) Find the derivative of $x^3 + 2x$, when c=1

$$\lim_{x \to 1} \left[\frac{f(x) - f(1)}{x-1}\right]$$

$$\lim_{x \to 1} \left[\frac{x^3+2x-3}{x-1}\right]$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 2 & -3 \\ & & 1 & 1 & 3 \\ \hline & 1 & 1 & 3 & \boxed{0} \end{array}$$

$$\lim_{x \to 1} \left[x^2 + x + 3\right] = 5$$

Sep 10-4:29 PM

ex) Find the eqn of the tangent and normal lines to $f(x) = (3+x)/x$, at the point when x=2

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x}{x} \cdot \frac{3+x+h}{x+h} - \frac{3+x}{x}\left(\frac{x+h}{x+h}\right)}{h}$$

$$\frac{\cancel{3x} + \cancel{x^2} + \cancel{hx} - \cancel{3x} - 3h - \cancel{x^2} - \cancel{hx}}{x(x+h)} \cdot \frac{1}{h}$$

$$\frac{-3h}{x(x+h)h} = \frac{-3}{x(x+h)}$$

$$f' = -\frac{3}{x^2}$$

$(2, 5/2)$ $f'(2) = -3/4 = m$

Tan $y - 5/2 = -3/4(x-2)$

Norm $y - 5/2 = 4/3(x-2)$

Sep 21-2:38 PM

p 102 # 7, 9, 19-25 odd, 31, 35, 37, 39-42,
45, 49, 61, 63, 71, 81, 90-92

Oct 3-10:02 AM