2.5 Implicit Differentiation

Explicit form of an equation is solved for y in terms of x.

ex) $y = 3x^2+5x-4$

Implicit form includes both x & y, and often cannot be solved for y in terms of x

ex) $3x - 4y + 5xy = 7x^2+2$

To take the derivative with respect to x (dy/dx), you must include y' in the equation.

ex1) $4y' + 6x = 7y^2 + 5x^3$

deriv: $4\cdot\frac{dy}{dx} + 6 = 14y\cdot\frac{dy}{dx} + 15x^2$

Then isolate y'

$$4\frac{dy}{dx} - 14y\cdot\frac{dy}{dx} = 15x^2-6$$

$$\frac{dy}{dx}(4-14y) = 15x^2-6$$

$$\boxed{\frac{dy}{dx} = \frac{15x^2-6}{4-14y}}$$

Oct 12-2:32 PM

2)Find the derivative of each with respect to x

a) $y^{1/2} = y^2x^3 + 5x + 2$

$$\tfrac{1}{2}y^{-1/2}\cdot y' = y^2(3x^2) + 2y\cdot y'(x^3) + 5$$

$$\tfrac{1}{2}y^{-1/2}y' - 2yx^3\cdot y' = 3y^2x^2 + 5$$

$$y'\left(\tfrac{1}{2}y^{-1/2} - 2yx^3\right) = 3y^2x^2+5$$

$$y' = \frac{3y^2x^2+5}{\frac{1}{2\sqrt{y}} - 2yx^3}$$

b) $y^3 = 2x - 3xy$

$$3y^2\cdot y' = 2 - [3x\cdot y' + 3y]$$

$$3y^2y' = 2 - 3xy' - 3y$$

$$3y^2y' + 3xy' = 2 - 3y$$

$$y' = \frac{2-3y}{3y^2+3x}$$

c) $\sin(y) = x$

$$\cos(y)\, y' = 1$$

$$y' = \frac{1}{\cos(y)}$$

d) $xy^2 = 7x + 3y - 9$

$$x\cdot 2yy' + 1y^2 = 7 + 3y'$$

$$2xy\cdot y' - 3y' = 7 - y^2$$

$$y' = \frac{7-y^2}{2xy-3}$$

Oct 15-3:10 PM

3) Find dy/dx and write the equation of the tangent line at (2,0)

$y^3 + y^2 - 5y - x^2 = -4$

$$3y^2y' + 2yy' - 5y' - 2x = 0$$

$$y'(3y^2+2y-5) = 2x$$

$$y' = \frac{2x}{3y^2+2y-5}$$

$$@(2,0)\quad y' = \frac{4}{-5} \rightarrow \text{slope}$$

$$y = -\frac{4}{5}x + \frac{8}{5}$$

Oct 15-3:13 PM

4) Find the slope of the graph of $2\sin(x)\cos(y)=1$, at point $(7\pi/6, 3\pi)$

$$2\cos(x)\cdot\cos y + -\sin(y)\cdot y'\cdot 2\sin x = 0$$

$$2\cos x\cdot\cos y = 2\sin x\cdot\sin y\cdot y'$$

$$y' = \frac{\cos x\cdot\cos y}{\sin x\cdot\sin y}$$

$$y' = \frac{\cos(7\pi/6)\cdot\cos(3\pi)}{\sin(7\pi/6)\cdot\sin(3\pi)}$$

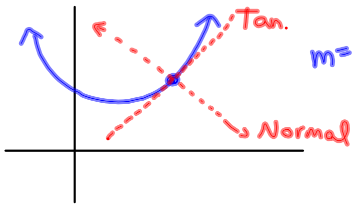
Tan: $x = 7\pi/6$

$$\frac{(-\sqrt{3}/2)\cdot(-1)}{(-1/2)(0)} = \text{und.}$$

vert. line

Oct 15-3:17 PM

Normal Lines -- perpendicular to the tangent line at a given point.



$m =$ Neg. Recip.

ex3) Find the tangent and normal lines to the graph of $x^2 + y^2 = 16$ at the point (0, -4)

$2x + 2y \cdot y' = 0$

$y' = \frac{-2x}{2y} = \frac{-x}{y}$

tan: $m = \frac{-0}{-4} = 0$

$\boxed{y = -4}$

Normal: $m =$ und.

$\boxed{x = 0}$

Oct 17-9:11 AM

hw: p142 # 3,7,11,13,15,23,27,29,31,37,43

Trig. Deriv. Quiz Tomorrow!

Oct 15-3:10 PM

Higher order implicit derivatives:

ex1) Find d^2y/dx^2 for: $x^2 - y^2 = 16$

$2x - 2yy' = 0$

$y' = \frac{x}{y}$

$y'' = \frac{y(1) - x \cdot y'}{y^2}$

$= \frac{y - x\left(\frac{x}{y}\right)}{y^2}$

$= \frac{y - \frac{x^2}{y}}{y^2} = \frac{\frac{y^2 - x^2}{y}}{y^2} \cdot \frac{1}{y^2} = \frac{y^2 - x^2}{y^3}$

$\frac{(x^2 - 16) - x^2}{y^3}$

$-16/y^3$

Oct 17-9:15 AM

2) Find the second derivative of $y^2 = x^3$

$2yy' = 3x^2$

$y' = \frac{3x^2}{2y}$

$y'' = \frac{2y(6x) - 3x^2(2 \cdot y')}{4y^2}$

$= \frac{12xy - 6x^2\left(\frac{3x^2}{2y}\right)}{4y^2}$ $\frac{12xy - \frac{9x^4}{y}}{4y^2}$

$\frac{\frac{12xy^2 - 9x^4}{y}}{4y^3}$

$\left(\frac{y}{y}\right) \frac{12yx - \frac{9x^4}{y}}{4y^2} = \boxed{\frac{12y^2x - 9x^4}{4y^3}}$

$y^2 = x^3$

$y = x^{3/2}$

$\frac{12x^3x - 9x^4}{4(x^{3/2})^3}$

$y'' = \frac{3\cancel{x^4}}{4\cancel{x^{9/2}}}$

Sep 22-3:15 PM

3) Find the slope of the normal line to the graph of $x^2 + 4y^2 = 4$ at the pt $(\sqrt{2}, -1/\sqrt{2})$

$2x + 8y\,y' = 0$

$8yy' = -2x$

$y' = \frac{-x}{4y}$

$y' = \frac{-\sqrt{2}}{4\left(\frac{-1}{\sqrt{2}}\right)} = 1/2$

Normal

$m = -2$

Oct 17-9:09 AM

Turn in tomorrow:
p 142

14*, 24, 26, 28, 32, 40, 44, 48, 50

*reduce answer using trig identites

Sep 22-3:17 PM