

2.6 Related Rates

-- derivatives with respect to *time t*
i.e. -- dy/dt

-- how fast something changes over time

ex1) area of a circle

$$A = \pi r^2$$

rate of change with respect to time:

$$dA/dt = 2\pi r (dr/dt)$$

If the radius of a circle grows 2 ft every hour,
what is the change in Area at the instant
when $r = 10$ feet?

$$A = \pi r^2$$

$$\begin{aligned} A' &= 2\pi r(dr/dt) \\ &= 2\pi(10)(2) = 40\pi \text{ ft}^2/\text{hr} \end{aligned}$$

ex2) $y = x^2 + 3$

Find change in y over time when $x = 1$,
given that change in x over time = 2

$$\begin{aligned}y' &= 2x(dx/dt) \\ &= 2(2)(1) = 4\end{aligned}$$

ex3) A pebble is dropped into a pond, causing circular ripples. The radius of the outer ripple is increasing at a constant rate of 1.5 feet per second. At what rate is the area of the outer circle changing when $r=4$ ft?

$$A = \pi r^2$$

$$\begin{aligned}A' &= 2\pi r(dr/dt) \\ &= 2\pi(4)(1.5) = 12\pi \text{ ft}^2/\text{sec}\end{aligned}$$

ex4) Air is pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find rate of change of the radius when $r=2$ in.

$$V = (4/3)\pi r^3$$

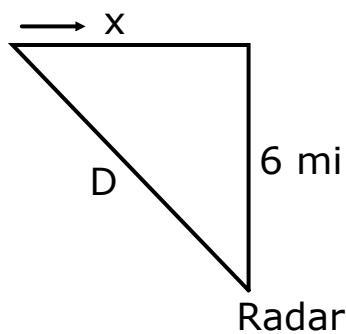
$$V' = 4\pi r^2(dr/dt)$$

$$4.5 = 4\pi(2^2)(dr/dt)$$

$$9/2 = 16\pi(dr/dt)$$

$$dr/dt = 9/(32\pi) \text{ in/min}$$

5) An airplane flies directly over a radar station at a height of 6 mi. The distance between the plane and the radar decreases at 400 mph. When the distance is 10 miles, what is the speed of the plane?



$$dx/dt = ?$$

$$dD/dt = -400 \text{ mph}$$

$$x^2 + 6^2 = D^2$$

$$2x(dx/dt) = 2D(dD/dt)$$

$$2(x)(dx/dt) = 2(10)(-400)$$

$$\text{when } D=10, x=8$$

b/c of Pythagorean Thm:

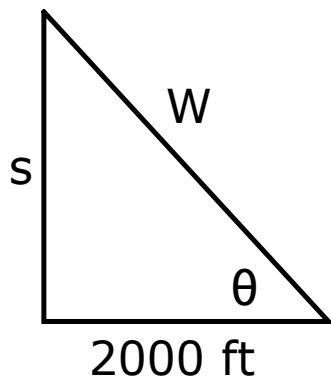
$$x^2 + 6^2 = 10^2$$

$$\text{Thus: } 2(8)(dx/dt) = 2(10)(-400)$$

$$dx/dt = -500 \text{ mph}$$

Speed of plane is 500 mph

- 6) A TV camera 2000 ft from a rocket's launch pad is filming the lift-off. The rocket rises vertically according to the position equation $s = 50t^2$, where s is in feet and t in seconds. Find the rate of change in the angle of elevation of the camera at 10 seconds after lift-off.



$$\tan \theta = s/2000 \quad , \quad ds/dt = 100t$$

$$\sec^2 \theta (d\theta/dt) = (1/2000)(ds/dt)$$

$$\sec^2 \theta (d\theta/dt) = (1/2000)(100t)$$

$$\text{when } t=10, \quad s=50(10^2) = 5000 \text{ ft}$$

$$\text{By Pyth Thm } w = \sqrt{29,000,000}$$

$$\text{thus } \sec \theta = \sqrt{(29000000)/2000}$$

$$(29000000/4000000)(d\theta/dt) = (1/2000)(1000)$$

$$(29/4)(d\theta/dt) = 1/2$$

$$d\theta/dt = 2/29 \text{ Rad/sec}$$

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$$14) d' = \frac{2x + \sin(2x)}{\sqrt{(x^2 + \sin^2 x)}}$$

$$16) A' = 2\pi r(dr/dt)$$

A' not constant since r is changing

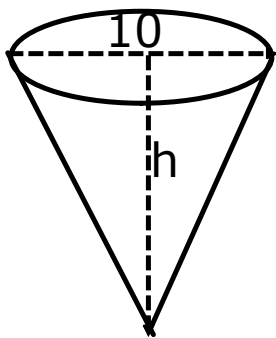
$$18a) 288\pi, 4608\pi \text{ in}^3/\text{min}$$

b) V' depends on r which is not constant

$$20a) 9 \text{ cm}^3/\text{sec}$$

$$b) 900 \text{ cm}^3/\text{sec}$$

- 2) A conical tank with vertex down is 10 feet across the top and 12 feet deep. If water is draining out of the tank at $10 \text{ ft}^3/\text{min}$, find the rate of change of the water depth when it is 8 feet deep.



$$V = (1/3)\pi r^2 h$$

$$\text{since } r/h = 5/12$$

$$r = (5/12)h$$

$$V = (1/3)\pi(5h/12)^2 h$$

$$V = 25\pi/(432)h^3$$

$$V' = (25\pi/144)h^2(dh/dt)$$

$$-10 = (25\pi/144)(8^2)(dh/dt)$$

$$dh/dt = -1440/(1600\pi) \text{ ft/min}$$