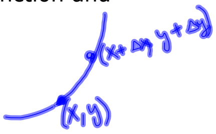
3.9 Differentials

Differential of y

Let $y = f(x)$ be a differentiable function and dx is a nonzero Real Number,

Then: $dy = f'(x) \cdot dx$



dy = differential of y
Δy = actual change in y = $f(x+\Delta x) - f(x)$

$\Delta y \approx dy$

Dec 5-9:10 AM

ex1) $y = x^2$

Find dy when x=1 and dx=.01

Compare this to the exact value of Δy

$dy = f'(x) \cdot dx$
$= 2x \cdot dx$
$= 2(1)(.01) = .02$

$\Delta y = f(x+\Delta x) - f(x)$
$f(1.01) - f(1)$
$(1.01)^2 - 1^2$
$1.0201 - 1 = .0201$

1b) Use the eqn of the tangent line to $f(x) = x^2$ at x=1 to approximate f(1.01)
This is the Linear approximation of f at x=1

Tan:
$y' = 2x$
$y' = 2(1) = 2$

$y = 2x + b$
$1 = 2(1) + b$
$-1 = b$

$y = 2x - 1$

$y = 2(1.01) - 1$
$= 2.02 - 1 = 1.02$

Dec 5-9:14 AM

a) Find dy for $y = 1 - 2x^2$ when $x = 0$ & $dx = -.1$

$dy = f'(x) \cdot dx$
$= -4x \cdot dx$
$= -4(0)(-.1) = 0$

when x=5
$dy = -4x \cdot dx$
$= -4(5)(-.1)$
$= 2$

Dec 5-9:14 AM

Error

Propagated Error

The amount a measurement (Δy) may be off due to an error in measurement (Δx)

% Error

The ratio of the error to the original measured value:

Relative Error = dy / y
Percent Error = Rel. Error * 100

ex3) A square with sides of length 4 cm may be off by at most $\pm.02$ cm when measured.

Find the propagated (actual) error and the percent error in calculating the Area of the square.

$A = x^2$
$dA = 2x \cdot dx$
$= 2(4)(\pm .02)$
$dA = \pm .16 \text{ cm}^2$

$dA/A = \frac{\pm .16}{16}$
$= \pm .01$
$\pm 1\%$

Dec 5-9:14 AM

ex4) Volume of a sphere:
The radius is 7 ft with a possible error of $\pm.05$ ft

Find the propagated & % error in volume

$V=\frac{4}{3}\pi r^3$

$dV= 4\pi r^2 \cdot dr$
$= 4\pi(7)^2(\pm.05)$
$= \pm 9.8\pi \text{ ft}^3$

$\frac{dV}{V} = \frac{\pm 9.8\pi}{\frac{4}{3}\pi 7^3}$
$= .0214$
$\pm 2.14\%$

Dec 5-9:15 AM

5) Use differentials to approximate $\sqrt{25.5}$

$y=\sqrt{x}$ $\quad x=25 \quad dx=.5$

$dy=\frac{1}{2}x^{-1/2}\cdot dx$
$=\frac{1}{2\sqrt{x}}dx$
$=\frac{1}{2\sqrt{25}}(.5) = \frac{1}{10}(.5) = \frac{1}{20}$

$\boxed{y=f(x)+dy}$

$y=\sqrt{25}+1/20$
$=\boxed{5.05}$

Dec 5-9:15 AM

6) if f(x) = sin(0.25πx), use differentials to approximate f(3.01)

$x=3 \quad dx=.01 \quad \sin\left(\frac{\pi}{4}x\right)$

$dy=\cos\left(\frac{\pi}{4}x\right)\cdot \pi/4 \cdot dx$
$=\cos 3\pi/4 \cdot \pi/4 \cdot (.01)$
$=\left(-\frac{1}{\sqrt{2}}\right)\pi/4\,(.01)$
$=-\frac{.01\pi}{4\sqrt{2}} = -\frac{\pi}{400\sqrt{2}}$

$y=f(x)+dy$
$=\sin(3\pi/4)+\frac{-\pi}{400\sqrt{2}}$
$=\frac{1}{\sqrt{2}}-\frac{\pi}{400\sqrt{2}}$

Nov 10-5:42 PM

p 233 # 5, 7, 17, 19, 23, 25, 33, 35, 36, 37, 38

Dec 5-9:59 AM