4.2 Summations & Area

Sigma Notation:

$$\sum_{i=1}^{n} a_i = a_1+a_2+\ldots+a_n$$

ex)

Nov 25-8:35 AM

Properties of Summation

1) $\sum ka_i = k \sum a_i$

2) $\sum(a_i \pm b_i) = \sum a_i \pm \sum b_i$

Summation Formulas:

1) $\sum_{i=1}^{n} c = cn$

2) $\sum_{i=1}^{n} i = n(n+1)/2$

3) $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$

4) $\sum_{i=1}^{n} i^3 = n^2(n+1)^2/4$

Nov 25-8:39 AM

ex) Evaluate $\sum (i+1)/n^2$

Nov 25-9:04 AM

Area of a Plane Region
--Upper and Lower Sums

ex) $f(x) = -x^2 + 5$, from x=0 to x=2

Nov 25-9:08 AM

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1-6, 15-19 odd

Nov 25-9:18 AM

For area on interval [a, b]

Left endpt: $m_i = a + (i-1)\Delta x$

Right endpt: $M_i = a + i\,\Delta x$

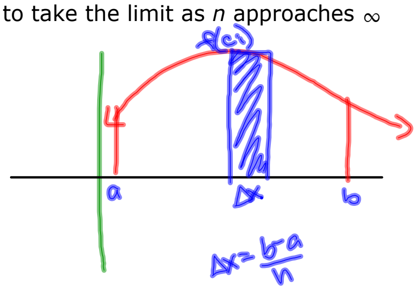
$\Delta x = \frac{b-a}{n}$ Rectangle width

Sums: $s(n) = \Sigma f(m_i)\Delta x$

$S(n) = \Sigma f(M_i)\Delta x$

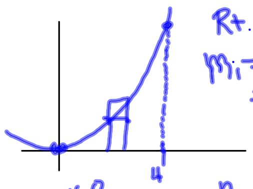
True Area $= \text{Lim}_{n\Rightarrow\infty} \Sigma f(c_i)\Delta x$

* You can choose either upper or lower sum to take the limit as *n* approaches ∞



Nov 25-9:13 AM

ex) Find the formulas for upper and lower sums for the region bounded by $f(x) = x^2$, $y = 0$, and $x = 4$.



Rt. endpt.

$m_i = a + i\Delta x$

$= 0 + i\frac{4}{n}$

$= \frac{4i}{n}$

$\Delta x = \frac{4-0}{n}$

$S(n) = \sum_{i=1}^{n} f(m_i)\cdot\Delta x$

$\Sigma f\left(\frac{4i}{n}\right)\cdot\left(\frac{4}{n}\right)$

$\Sigma \left(\frac{4i}{n}\right)^2\cdot\left(\frac{4}{n}\right)$

$\Sigma \frac{16i^2}{n^2}\cdot\frac{4}{n}$

$= \frac{64}{n^3}\Sigma i^2$

$S(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

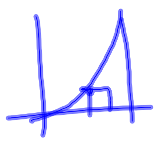
$\lim_{n\to\infty} S(n) = \frac{128}{6} = 64/3$

b) Find the limit of each sum as n approaches ∞

Lower Sum

$\lim_{n\to\infty} \frac{32(2n^3-3n+1)}{3n^3} = 64/3$

Dec 1-8:29 AM



Left endpt.

$s(n)$

$m_i = a + (i-1)\Delta x$

$= (i-1)\frac{4}{n}$

$s(n) = \sum_{i=1}^{n} f(m_i)\cdot\Delta x$

$\Sigma (i-1)^2\frac{16}{n^2}\cdot\frac{4}{n}$

$\frac{64}{n^3}\Sigma i^2 - 2i + 1$

$\frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6} - \frac{\cancel{2}\,n(n+1)}{\cancel{2}} + n\right]$

$\frac{128}{6} + 0 + 0$

Jan 5-11:58 AM

Upper Sum

Rt Endpt

$m_i = a + i\Delta x$

$= 0 + i\left(\frac{4}{n}\right) = \frac{4i}{n}$

$S(n) = \sum_{i=1}^{n} f(m_i)\cdot \Delta x$

$= \sum f\left(\frac{4i}{n}\right)\cdot \frac{4}{n}$

$= \sum \frac{16i^2}{n^2}\cdot \frac{4}{n}$

$= \frac{64}{n^{\cancel{3}2}}\left[\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$

$= \frac{64}{n^2}\left[\frac{2n^2+3n+1}{6}\right]$

$= \frac{32(2n^2+3n+1)}{3n^2}$

$\lim_{n\to\infty} S(n) = \frac{64}{3}$

Dec 1-10:43 AM

ex) find the area of the region bounded by $f(x) = x^3$, and the x-axis from 0 to 1

Dec 1-8:49 AM

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31-35, 39-43 odd

49, 51, 59

Dec 1-8:51 AM