

4.4 Fundamental Thm of Calculus

Let $f(x)$ be integrable on $[a,b]$, and $F(x)$ be the antiderivative of $f(x)$, then:

$$\int_a^b f(x) dx = F(b) - F(a)$$

The absolute value of this quantity is the area between the curve and the x-axis from a to b .

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ex1) Find the area between $f(x)$ and the x-axis from $x=1$ to $x=4$.

Let $f(x) = \frac{x + \sqrt{x}}{2}$

$$\int_1^4 \left[\frac{x + \sqrt{x}}{2} \right] = \frac{1}{2} \left[\frac{x^2}{2} + \frac{2}{3} x^{3/2} \right]_1^4$$

$$\frac{1}{2} \left[\left(8 + \frac{2}{3} \cdot 8 \right) - \left(\frac{1}{2} + \frac{2}{3} \right) \right]$$

$$\frac{1}{2} \left[8 + 16/3 - 1/2 - 2/3 \right]$$

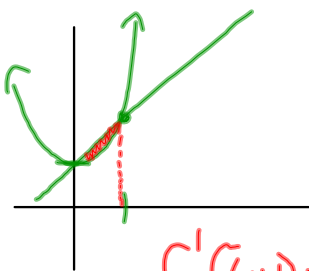
$$\frac{1}{2} \left[15/2 + 14/3 \right]$$

$$45/6 + 28/6$$

$$\frac{73}{12}$$

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2) Sketch and find the area bounded by $f(x) = x + 1$, and $g(x) = x^2 + 1$



$$x+1 = x^2+1$$

$$0 = x^2 - x$$

$$0 = x(x-1)$$

$$\int_0^1 [(x+1) - (x^2+1)]$$

$$\int_0^1 [x - x^2]$$

$$\left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \left(\frac{1}{2} - \frac{1}{3} \right) - (0)$$

$$= \boxed{\frac{1}{6}}$$

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- To find area between 2 curves:
- 1) Find all points of intersection
 - 2) Sketch graph of region
 - 3) Integrate the upper curve minus the lower curve between points of intersection.

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4.4 (continued)

Mean Value Thm for Integrals

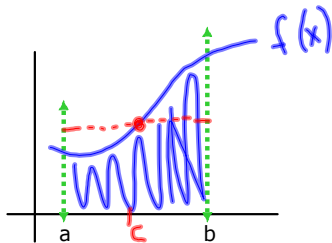
If $f(x)$ is continuous on $[a,b]$, then there exists a number c in the interval such that:

$$\int_a^b f(x) dx = f(c)(b-a)$$

Average Value of a Function on an Interval

If $f(x)$ is integrable on $[a,b]$, then the average value of f on the interval is:

$$\frac{1}{b-a} \int_a^b f(x) dx = f(c)$$



ex) Find the avg value of $f(x) = 3x^2 - 2x$ on $[1,4]$, then find all x -values where f equals its avg value.

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Second Fundamental Thm of Calculus

If $f(x)$ is continuous on an open interval containing 'a', then for every x in the interval:

$$\frac{d}{dx} \left[\int_a^x f(t) dt \right] = f(x)$$

*** This thm states that if you integrate a function, then take the derivative of it, the result is the original function.

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2, 12, 14, 22, 32
38, 40, 42, 46, 50
54-60

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