4.5

Integration by Substitution

$$\int f(g(x))\, g'(x)\, dx$$

Let $u = g(x)$ & $du = g'(x) \cdot dx$

then $= \int f(u) \cdot du$

ex1) $\int (x^2+1)^4 (2x)\, dx$

$u = x^2+1$

$du = 2x \cdot dx$

$= \int u^4 \cdot du$

$= \frac{u^5}{5} + C$

$= \frac{(x^2+1)^5}{5} + C$

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2) $\int 3x^2 \sqrt{x^3+1} \cdot dx$

$u = x^3+1$

$du = 3x^2 \cdot dx$

$= \int \sqrt{u} \cdot du$

$= \int u^{1/2}\, du$

$= \frac{2u^{3/2}}{3} + C$

$= \boxed{\frac{2(x^3+1)^{3/2}}{3} + C}$

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3) $\frac{1}{2}\int 2x\, (x^2+1)^3 \cdot dx$

$u = x^2+1$

$du = 2x \cdot dx$

$= \frac{1}{2}\int 2x (x^2+1)^3 \cdot dx$

$= \frac{1}{2}\int u^3 \cdot du$

$= \frac{1}{2}\left[\frac{u^4}{4}\right] + C$

$\frac{1}{2} \cdot \frac{(x^2+1)^4}{4} + C$

$\boxed{\frac{(x^2+1)^4}{8} + C}$

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4) $\frac{1}{4}\int 4x^3 \sqrt{x^4+3} \cdot dx$

$u = x^4+3$

$du = 4x^3 \cdot dx$

$= \frac{1}{4}\int 4x^3 \sqrt{x^4+3}\, dx$

$= \frac{1}{4}\int u^{1/2} \cdot du$

$\frac{1}{4}\left[\frac{2u^{3/2}}{3}\right] + C$

$\frac{(x^4+3)^{3/2}}{6} + C$

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Substitution w/ Intervals

$[a, b]$

$$\int_a^b f(u)\,du = \int_{g(a)}^{g(b)} f(u)\,du$$

$u = g(x)$

ex1) $\int_0^1 x(x^2+1)^2\,dx$

$u = x^2+1$

$du = 2x \cdot dx$

$$\frac{1}{2}\int_0^1 2x(x^2+1)^2\,dx$$

$$\frac{1}{2}\int_1^2 u^2 \cdot du$$

$x = 0 \rightarrow u = 0^2+1$

$x = 1 \rightarrow u = 1^2+1$

$$\frac{1}{2}\left[\frac{u^3}{3}\right]_1^2$$

$$\left[\frac{u^3}{6}\right]_1^2 = \frac{8}{6} - \frac{1}{6} = \frac{7}{6}$$

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$$\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$$

$u = 2x+1$

$du = 2 \cdot dx$

$$\frac{1}{2}\int_0^4 \frac{2}{\sqrt{2x+1}}\,dx$$

$$\frac{1}{2}\int_1^9 \frac{du}{\sqrt{u}}$$

$$\frac{1}{2}\int_1^9 u^{-1/2}\,du$$

$$\cancel{\frac{1}{2}}\left[\cancel{2}u^{1/2}\right]_1^9 = \sqrt{9} - \sqrt{1} = 2$$

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AP Calc HW:

p 297 # 1-12, 17, 21, 25, 35, 37, 39, 40-43

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