4.6 Numeric Integration

Some functions do not have an antiderivative that is elementary.
(ie - it cannot be found with basic integration)

Therefore, we must use numeric approximations.

ex) $\int \sqrt{1+x^2}\cdot dx$

2 Methods of Numeric Approximation for Integrals:

#1) Trapezoidal Rule

#2) Simpson's Rule

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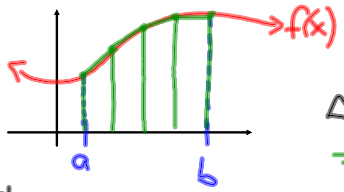
#1) Trapezoidal Rule

For a function f(x) that is continuous on [a, b], we split the interval into "n" trapezoids.



$\Delta x = \frac{b-a}{n}$
= width of Trap.

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + f(x_n)\right]$$

As $n \to \infty$, this will approach the true area.

Notice the coefficients follow the pattern:
1, 2, 2, 2, ... 2, 2, 1

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Ex 1) Trap. Rule

$\int_0^1 \sqrt{1+x^2}\cdot dx$, for $n=4$

$\Delta x = 1/4$

$$A = \frac{1-0}{2(4)}\left[f(0) + 2f\left(\tfrac{1}{4}\right) + 2f\left(\tfrac{1}{2}\right) + 2f\left(\tfrac{3}{4}\right) + f(1)\right]$$

$$= \frac{1}{8}\left[1 + \cancel{2}\left(\frac{\sqrt{17}}{\cancel{4}\,2}\right) + \cancel{2}\left(\frac{\sqrt{5}}{\cancel{2}}\right) + \cancel{2}\left(\frac{5}{\cancel{4}\,2}\right) + \sqrt{2}\right]$$

$A \approx 1.151$

True Area ≈ 1.148

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#2) Simpson's Rule

For a function f(x) that is continuous on [a,b], we use quadratics to approximate the area. For this rule, "n" must be an even integer.

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)\right]$$

ex2) $\int_0^1 \sqrt{1+x^2}\cdot dx$, for $n=4$

pattern: 1, 4, 2, 4, 2, ... 4, 1

$$A = \frac{1}{12}\left[f(0) + 4f\left(\tfrac{1}{4}\right) + 2f\left(\tfrac{1}{2}\right) + 4f\left(\tfrac{3}{4}\right) + f(1)\right]$$

$$\frac{1}{12}\left[\sqrt{1} + 4\sqrt{1+\tfrac{1}{16}} + 2\sqrt{1+\tfrac{1}{4}} + 4\sqrt{1+\tfrac{9}{16}} + \sqrt{2}\right]$$

≈ 1.1478 Trap 1.1515

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p305 #11, 13, 15

Turn In Tues!
p307 #4, 8, 10, 12, 18, 36, 72

Jan 15-12:04 PM