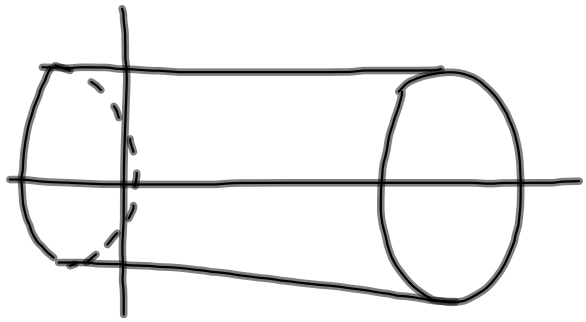
6.3 Shell Method

P 432



Horizontal Axis

$$V = 2\pi \int_c^d p(y) \cdot h(y) \cdot dy$$

Vertical Axis

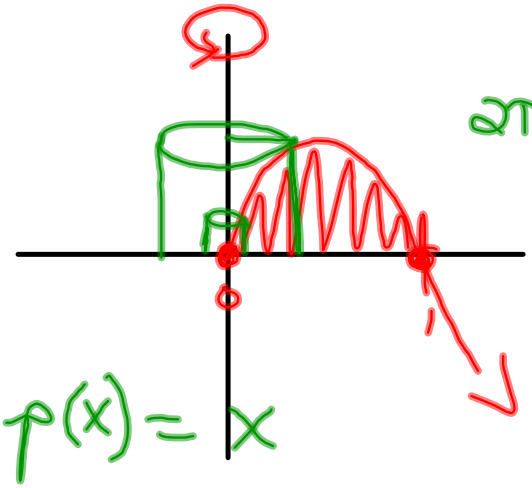
$$V = 2\pi \int_a^b p(x) \cdot h(x) \cdot dx$$

$p(x) \rightarrow$ distance from the shell to the axis of Revolution (Radius)

$h(x) \rightarrow$ height of the shell

ex1) $y = x - x^3$, $y = 0$, $x = 0$

Revolve around y-axis



$p(x) = x$

$h(x) = y = x - x^3$

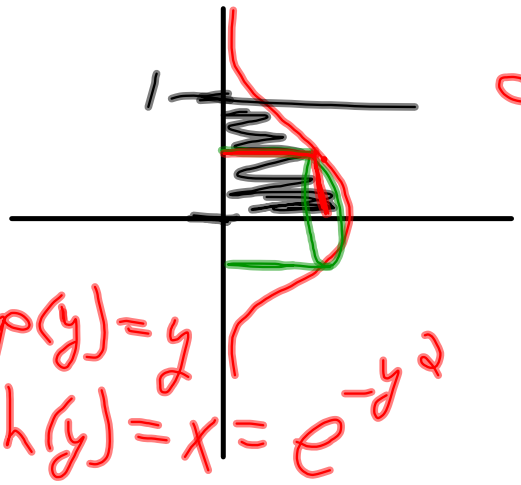
$$2\pi \int_0^1 (x)(x - x^3)\,dx$$

$$= \frac{4\pi}{15}$$

2) $x = e^{-y^2}$, $x = 0$

from $0 \le y \le 1$

Revolve around x-axis



$r(y) = y$

$h(y) = x = e^{-y^2}$

$$2\pi \int_0^1 (y)\, e^{-y^2}\, dy$$

$u = -y^2$

$du = -2y\, dy$

$$-\frac{1}{2} 2\pi \int e^u\, du$$

$$-\pi \left[e^u \right]_0^{-1}$$

$$-\pi \left[e^{-1} - e^0 \right]$$

$$\pi \left[1 - \frac{1}{e} \right]$$

ex3) Find the volume when the region bounded by $y=x\sqrt{x+1}$ and $y=0$ is revolved around the y-axis.

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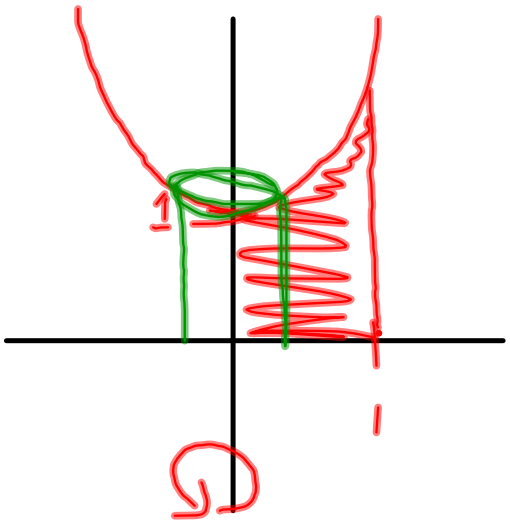
ex4) Find the volume of the solid formed by revolving the region bounded by $y=x^2+1$, $x=0$, $y=0$, and $x=1$ about the y-axis.



$p = x$

$h = x^2+1$

$$2\pi \int_0^1 x(x^2+1)\,dx$$

$$= 2\pi \int_0^1 x^3 + x$$

$$= 3\pi/2$$

ex5) Find the volume of the solid formed by revolving the region bounded by $y=x^3+x+1$, $y=1$, and $x=1$ about the line $x=2$.

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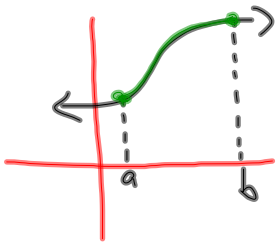
hw: p437 # 3,4,9,15,16,19,26

6.4 Arclength & Surface Area



Arc Length (s) → the distance along the curve.

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$s = \int_c^d \sqrt{1 + [f'(y)]^2}\, dy$$

ex) Find arc length of $y = \frac{x^3}{6} + \frac{1}{2x}$ from $[\frac{1}{2}, 2]$

$$s = \int_{\frac{1}{2}}^{2} \sqrt{1 + (y')^2}\, dx$$

$y = \frac{x^3}{6} + \frac{1}{2}x^{-1}$

$y' = \frac{x^2}{2} - \frac{1}{2}x^{-2}$

$$= \int_{\frac{1}{2}}^{2} \sqrt{1 + \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2}$$

$$= \int_{\frac{1}{2}}^{2} \sqrt{1 + \frac{x^4}{4} -}$$

$$= \int_{\frac{1}{2}}^{2} \left[\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right]$$

$$= \left[\frac{1}{2}\cdot\frac{x^3}{3} + \frac{1}{2}\cdot\frac{x^{-1}}{-1}\right]_{\frac{1}{2}}^{2}$$

$$\left[\frac{x^3}{6} - \frac{1}{2x}\right]_{\frac{1}{2}}^{2} = \left(\frac{4}{3} - \frac{1}{4}\right) - \left(\frac{1}{48} - 1\right) = \frac{33}{16}$$

Surface Area

→ outside area of a shape

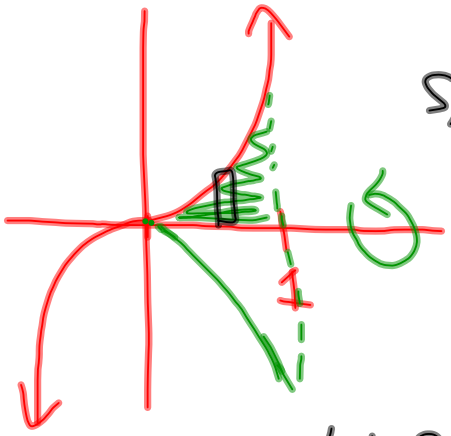
$$SA = 2\pi \int_a^b r(x)\sqrt{1+[f'(x)]^2}\cdot dx$$

$$SA = 2\pi \int_c^d r(y)\sqrt{1+[f'(y)]^2}\cdot dy$$

ex2) Surface area of

$y' = 3x^2$ $f(x) = x^3$ from $[0,1]$

Revolve around x-axis



$$SA = 2\pi \int_0^1 (x^3)\sqrt{1+(3x^2)^2}$$

$$= \frac{1}{36}\cdot 2\pi \int_0^1 36x^3\sqrt{1+9x^4}\cdot dx$$

$u = 1+9x^4$

$du = 36x^3\cdot dx$

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