P.3 Functions & Graphs

Relation -- a set of ordered pairs (x, y)

Function -- a relation in which each member of the domain (x) is paired with *exactly one* member of the range (y).

-- must pass the Vertical Line Test in which all vert. lines can hit the graph no more than once

Implicit form: $x^2 + 2y = 1$
Explicit form: $y = 1/2(1 - x^2)$
Function Notation: $f(x) = 1/2(1 - x^2)$

Denom $\neq 0$
$\sqrt{\quad} \geq 0$

Domain (x) -- the independent variable
Range (y) -- the dependent variable, also called the "image" of x under *f*

State the domain of the following functions:

1) $f(x) = \frac{1}{x^2 - 4}$

$x \neq \pm 2$

All R except ± 2
$(-\infty,-2)(-2,2)(2,\infty)$

2) $g(x) = \sqrt{x - 7}$

$x - 7 \geq 0$
$x \geq 7$
All R ≥ 7
$[7,\infty)$

3) $h(x) = 2x - 4x^3 + 8$

All R
$(-\infty,\infty)$

4) $j(x) = \frac{x + 3}{\sqrt{x + 2}}$

All R > -2
$(-2,\infty)$

ex) $f(x) = \frac{\sqrt{5-x}}{x^2-36}$

$x \neq \pm 6$ $(-\infty,-6)(-6,5]$

$5 - x \geq 0$
$5 \geq x$
$x \leq 5$

5) Let $f(x) = x^2 + 2x - 7$, calculate the following:

f(3a) = $9a^2 + 6a - 7$

f(b - 1) = $b^2 - 8$

Difference Quotient

-- a formula for the slope of any curve at a point.

$$= \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

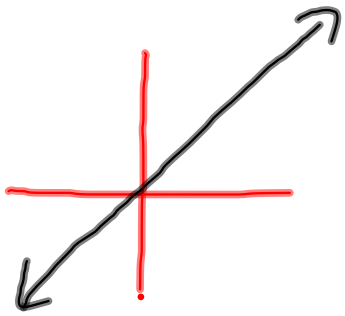
or:

$$= \frac{f(x + h) - f(x)}{h}, \quad \text{where } h = \Delta x$$

1) $f(x) = 7x - 4$

2) $g(x) = x^2 + 3x - 5$, find the slope when $x = 2$

One-to-One Function -- each x is paired with exactly one y,
and each y is paired with exactly one x
Must pass both vertical & horizontal line tests.



$y = x$

$y = x^3$

Transformations on f(x)		ex: $f(x) = x^2$
-- Horizontal Shift Right:	$f(x - c)$	$(x - 3)^2$
-- Horizontal Shift Left:	$f(x + c)$	$(x + 4)^2$
-- Vertical Shift Up:	$f(x) + c$	$x^2 + 7$
-- Vertical Shift Down:	$f(x) - c$	$x^2 - 2$
-- Reflect across x-axis:	$- f(x)$	$- x^2$
-- Reflect across y-axis:	$f(-x)$	$(-x)^2$
-- Reflect through origin:	$- f(-x)$	$-(-x)^2$

ex1: $f(x) = \sin x$

$g(x) = 5 + \sin(x + 1)$

H left 1
V up 5

$h(x) = -\sin(x - 2)$

Reflect x
H. Rt 2

ex2: $f(x) = x^3$

reflect across x-axis, shift left 3, then shift down 7

$g = -(x+3)^3 - 7$

Elementary Functions

1) Algebraic Functions
   --polynomials $(x^3 - 5x^2 + 2x - 7)$
   --radical $(\sqrt{7 - 5x})$
   --rational $\left(\frac{x + 3}{6x^2}\right)$

2) Trigonometric Functions
   --sin(x), cos(x), tan(x), etc...

3) Exponential & Logarithmic Functions
   -- e^x, ln(x)

Composite Functions

$f \circ g(x) = f(g(x))$ "f of g"

$g \circ f(x) = g(f(x))$ "g of f"

1) $f(x) = x^2 - 3x + 4$, $g(x) = x + 1$

$f \circ g(x) =$ $g(f(-2)) =$

$(x+1)^2 - 3(x+1) + 4$

$x^2 - x + 2$

$f(-2) = 14$

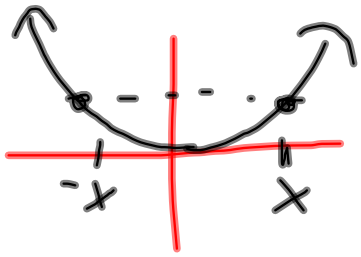
$g(14) = 15$

Even Functions -- y-axis symmetry

-- f(x) = f(-x)

-- degree of x is even

ex:

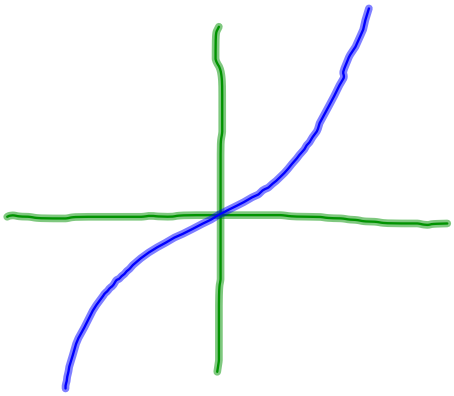


Odd Functions -- origin symmetry

-- f(x) = -f(-x)

-- degree of x is odd

ex:



HW: p27, # 4 -18e, 49, 52-56, 59-62 turn in Mon.