1.3 Evaluating Limits Analytically

Well-Behaved Functions -- are continuous at 'c'.
-- solve with direct substitution .

$$\lim_{x \to c} f(x) = f(c)$$

ex) $\lim_{x \to 3} \dfrac{x^2 + 2x - 8}{x + 7} = \dfrac{3^2+6-8}{3+7} = 7/10$

Basic Limits:
Let *b* and *c* be real numbers and let *n* be a positive integer.

1. $\lim_{x \to c} b = b$ 2. $\lim_{x \to c} x = c$ 3. $\lim_{x \to c} x^n = c^n$

ex1) $\lim_{x \to 8} 3 = 3$ 2) $\lim_{x \to 9} x = 9$ 3) $\lim_{x \to -4} x^2 = 16$

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the given limits:

$$\lim_{x \to c} f(x) = L \qquad \lim_{x \to c} g(x) = K$$

1. Scalar Multiple $\quad \lim_{x \to c} [b\, f(x)] = bL$
2. Sum/Difference $\quad \lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product $\quad \lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient $\quad \lim_{x \to c} [f(x)/g(x)] = L/K,\ K \neq 0$
5. Power $\quad \lim_{x \to c} [f(x)]^n = L^n$

ex) $\lim f(x) = 3$

$\lim g(x) = 10$

a) $\lim f(x)^5 = 3^5$

b) $\lim [f - g] = 3 - 10 = -7$

c) $\lim 40\, g(x) = 400$

Evaluating Limits:

Cancellation -- find an equivalent function by factoring and simplifying the equation.

ex) $\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} = \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}}$

$= x - 2$

$= (-3) - 2 = -5$

Rationalization -- limits involving a square root binomial can be rationalized by multiplying by the conjugate.

ex) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} \left(\frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}\right)$

$$= \frac{x+1-1}{x(\sqrt{x+1}+1)}$$

$$= \frac{x}{x(\sqrt{x+1}+1)}$$

$$= \frac{1}{\sqrt{x+1}+1}$$

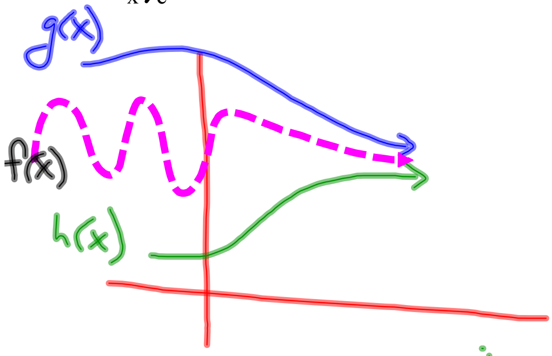
$$= \frac{1}{\sqrt{0+1}+1} = \frac{1}{2}$$

Squeeze Theorem

If $h(x) < f(x) < g(x)$ for all x in an open interval containing c, and if

$$\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$$

then $\lim_{x \to c} f(x)$ exists and $= L$



ex #77) $c = 0$ $\quad 4 - x^2 \le f(x) \le 4 + x^2$

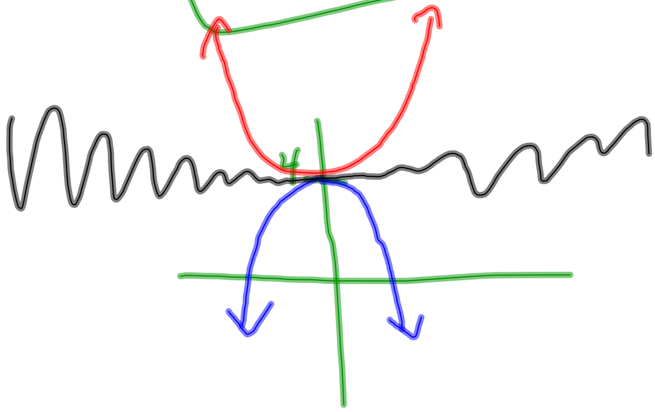
$$\lim_{x \to c} f(x)$$

$$\lim_{x \to 0} f(x)$$

$$\lim_{x \to 0} (4 - x^2) = 4$$

$$\lim_{x \to 0} (4 + x^2) = 4$$

$$\boxed{\lim_{x \to 0} f(x) = 4}$$



1.3 Review ex's

① $\lim_{x \to c} f(x) = 4$

ⓐ $\lim_{x \to c} \sqrt[4]{f(x)} = \sqrt[4]{4} = 4\text{^}(1/4)$

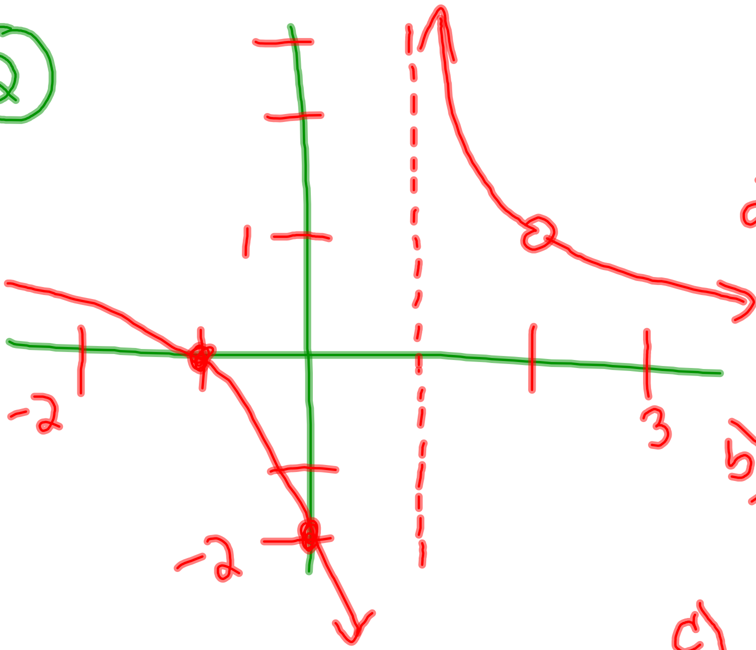
≈ 1.414

ⓑ $\lim_{x \to c} \left[\frac{f(x)^2}{3} - 7 \right]$

$= \frac{16}{3} - 7\left(\frac{3}{3}\right)$

$= -5/3$

②



a) $\lim_{x \to 2} f(x) = 1$

b) $\lim_{x \to 1} f(x) =$ DNE

c) $\lim_{x \to 0} f(x) = -2$

d) $\lim_{x \to -1} f(x) = 0$

③ $$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 3(x+\Delta x) + 2 - (x^2 - 3x + 2)}{\Delta x}$$

$$\frac{\cancel{x^2} + h^2 + 2xh - \cancel{3x} - 3h + \cancel{2} - \cancel{x^2} + \cancel{3x} - \cancel{2}}{h}$$

$$\frac{h^2 + 2xh - 3h}{h}$$

$$\lim_{h \to 0} (h + 2x - 3) = \boxed{2x - 3}$$

Free-Falling Objects:

Position above ground is given by the function
$s(t) = -16t^2 + h_o$

Velocity of the object is given by the function
$$V(t) = \lim_{t \to a} \frac{s(a) - s(t)}{a - t}$$

a = time you approach

ex1) If you drop an object from a height of 20,000 ft, how fast will it be falling after 5 sec?

$s(t) = -16t^2 + 20000$

a

$s(5) = 19600$

$$V = \lim_{t \to 5} \frac{s(5) - s(t)}{5 - t}$$

$$\lim_{t \to 5} \frac{19600 + 16t^2 - 20000}{5 - t}$$

$$\frac{16t^2 - 400}{5 - t}$$

$$\frac{16(t^2 - 25)}{5 - t}$$

$$\lim_{t \to 5} \frac{16(t+5)\overset{(-1)}{\cancel{(t-5)}}}{\cancel{5-t}}$$

$= -16(5+5) = -160$ ft/sec

hw1: p 64 # 5-17 odd, 29, 31, 41-51odd, 75, 77 ✓

Do in HW Notebook

HW2: Turn in Fri

p 64 # 32-50e, 74, 78