

2.3 Product & Quotient Laws

Product Law

-- allows you to take the derivative of
2 functions that are multiplied

$$\frac{d}{dx} [f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

ex1) $y = (x^3 - 3x + 4)(5x^4 - 3)$

$$\begin{aligned} y' &= (x^3 - 3x + 4)(20x^3) + (5x^4 - 3)(3x^2 - 3) \\ &= 20x^6 - 60x^4 + 80x^3 + 15x^6 - 15x^4 - 9x^2 + 9 \\ &= 35x^6 - 75x^4 + 80x^3 - 9x^2 + 9 \end{aligned}$$

ex2) $y = \frac{5}{7x^2} \cdot (x^3 - 5x)$

$$\begin{aligned} y &= \frac{5}{7} x^{-2} \cdot (x^3 - 5x) \\ y' &= \frac{5}{7} x^{-2} \cdot (3x^2 - 5) + (x^3 - 5x) \left(\frac{-10}{7} x^{-3} \right) \\ &= \frac{5(3x^2 - 5)}{7x^2} - \frac{10(x^3 - 5x)}{7x^3} \\ &= \frac{5x(3x^2 - 5) - 10(x^3 - 5x)}{7x^3} \\ &= \frac{15x^3 - 25x - 10x^3 + 50x}{7x^3} \\ &= \frac{5x^3 + 25x}{7x^3} \\ &= \frac{5x^2 + 25}{7x^2} \end{aligned}$$

ex) $f(x) = (4x+1) \sqrt{x}$

$$f = (4x+1) \cdot x^{1/2}$$

$$f' = (4x+1) \cdot \frac{1}{2} x^{-1/2} + x^{1/2} \cdot (4)$$

$$= \frac{4x+1}{2\sqrt{x}} + 4\sqrt{x} \left(\frac{2\sqrt{x}}{2\sqrt{x}} \right)$$

$$\begin{array}{l} 2x^{1/2} + \frac{1}{2}x^{-1/2} + 4x^{1/2} \\ 6\sqrt{x} + \frac{1}{2\sqrt{x}} \end{array} \left| \begin{array}{l} \frac{4x+1}{2\sqrt{x}} + \frac{8x}{2\sqrt{x}} \\ \frac{12x+1}{2\sqrt{x}} \end{array} \right.$$

Quotient Law -- allows you take the derivative of 2 functions that are divided

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$\frac{\text{bottom} \cdot \text{top}' - \text{top} \cdot \text{bottom}'}{\text{bottom}^2}$$

ex3) $f(x) = \frac{5x}{x^2 + 7}$

$$f' = \frac{(x^2+7) \cdot 5 - (5x) \cdot (2x)}{(x^2+7)^2}$$

$$f' = \frac{-5x^2 + 35}{(x^2+7)^2} = \frac{-5(x^2-7)}{(x^2+7)^2}$$

ex4) Find the eqn of the tangent line to $y = (4x)/(x^2+3x-5)$ at $x = -1$

$$y = \frac{4x}{x^2+3x-5} \quad (-1, 4/7)$$

$$y' = \frac{(x^2+3x-5) \cdot 4 - (4x) \cdot (2x+3)}{(x^2+3x-5)^2}$$

$$= \frac{4x^2 + 12x - 20 - 8x^2 - 12x}{(\quad)^2}$$

$$y' = \frac{-4x^2 - 20}{(x^2+3x-5)^2}$$

$$y'(-1) = \frac{-4 - 20}{(1 - 3 - 5)^2} = \frac{-24}{49} \quad (-1, 4/7)$$

$$y = -\frac{24}{49}x + b$$

$$4/7 = -\frac{24}{49}(-1) + b$$

$$b = 4/7 - 24/49$$

$$b = 28/49 - 24/49 = 4/49$$

$$y = -\frac{24}{49}x + 4/49$$

$$49y = -24x + 4$$

p121 1-4, 7-25odd, 51,53,57, 59-62, 65,67,83