3.1 Extrema on an Interval

Absolute Min/Max

1. $f(c)$ is the **minimum** of f on an interval if $f(c) \le f(x)$ for all x in the interval.

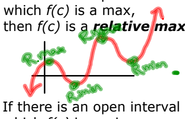
2. $f(c)$ is the **maximum** of f on an interval if $f(c) \ge f(x)$ for all x in the interval.

Extreme Value Theorem

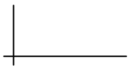
If f is continuous on a closed interval [a,b], then f has both a min & max on the interval.

Relative Extrema

1. If there is an open interval containing c on which $f(c)$ is a max, then $f(c)$ is a ***relative maximum***.



2. If there is an open interval containing c on which $f(c)$ is a min, then $f(c)$ is a ***relative minimum***.

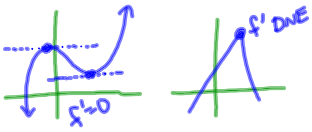


Critical Number

Let f be defined at c. If $f'(c) = 0$ or if f is **not** differentiable at c, then c is a critical number of f.

$f'(c) = 0$ or DNE

**Relative Extrema occur *only* at critical numbers



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To find extrema on [a,b]:

1. find all critical numbers in (a,b) $f' = 0$, or DNE
2. Evaluate f at each critical number (find y)
3. Evaluate f at each endpoint of [a,b] $f(a), f(b)$
4. Locate min & max value

ex1) Find extrema of $f(x) = 3x^4 - 4x^3$ on [-2,2]

$f' = 12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

c.p.: $x = 0, 1$ $\quad f = 3x^4 - 4x^3$

$f(0) = 0$

$f(1) = -1$

$f(-2) = 3(16) - 4(-8) = 48 + 32 = 80$

$f(2) = 3(16) - 4(8) = 48 - 32 = 16$

min @ (1,-1) $\quad$ max @ (-2,80)

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ex2) Find extrema of $f(x) = 2x - 3x^{2/3}$ on [-1,3]

$f' = 2 - 2x^{-1/3}$

$= 2 - \frac{2}{\sqrt[3]{x}} = 0$

$\frac{2}{1} = \frac{2}{\sqrt[3]{x}}$

$2\sqrt[3]{x} = 2$

$(\sqrt[3]{x} = 1)^3$

$x = 1$

c.p. $x = 0, 1$

$2x - 3x^{2/3}$

$f(x) = 2x - 3\sqrt[3]{x^2}$,

$f(0) = 0$

$f(1) = -1$

$f(-1) = 2(-1) - 3(1) = -5$

$f(3) = 6 - 3\sqrt[3]{9}$

$\boxed{\begin{array}{l} \min(-1,-5) \\ \max(0,0) \end{array}}$

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hw p 160 #1-10, 15-23 odd, 29-32

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