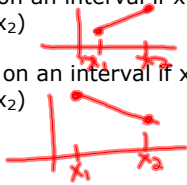


3.3 Increasing & Decreasing Functions & First Derivative Test

A function f is increasing on an interval if $x_1 < x_2$ implies that $f(x_1) < f(x_2)$

A function f is decreasing on an interval if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$



First Derivative Test for Inc./Dec.

- $f'(x) > 0$ on (a,b) , then $f(x)$ is increasing
- $f'(x) < 0$ on (a,b) , then $f(x)$ is decreasing
- $f'(x) = 0$ on (a,b) , then $f(x)$ is constant (horiz)



Finding Intervals for Inc./Dec.

- Find all critical #'s c (where $f'(x)=0$ or undef.)
- Set up intervals using all c values
- Find $f'(x)$ for one test value in each interval:
 $f'(x) > 0$ Inc
 $f'(x) < 0$ Dec

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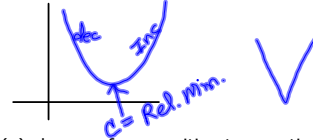
Strictly Monotonic -- functions that are always increasing or always decreasing.

ex) $f(x) = x$ $g(x) = -x^3$

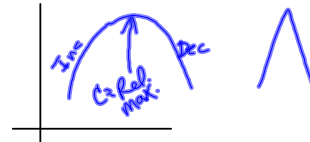


First Derivative Test

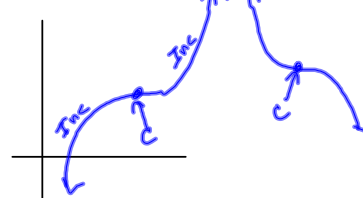
- If $f'(x)$ changes from negative to positive at c , then $f(c)$ is a Relative Minimum



- If $f'(x)$ changes from positive to negative at c , then $f(c)$ is a Relative Maximum



Neither min or max:



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Find all rel. extrema & Inc/Dec intervals

ex 1) $f(x) = x^3 - 3/2 x^2$

$f' = 3x^2 - 3x = 0$
 $3x(x-1) = 0$
 C.P: $x=0, 1$
 $f(-\infty, 0)$ $f' = \text{Pos}$ Inc
 $(0, 1)$ $f' = \text{Neg}$ Dec
 $(1, \infty)$ $f' = \text{Pos}$ Inc
 max $(0, 0)$
 min $(1, -1/2)$

2. $f(x) = (x^2 - 4)^{2/3}$

$f' = \frac{2}{3}(x^2 - 4)^{-1/3} \cdot (2x)$
 $= \frac{4x}{3\sqrt[3]{x^2 - 4}} = 0$
 $4x = 0$ $x = 0$
 $x^2 - 4 = 0$ $x = \pm 2$
 C.P: $x = \pm 2, 0$
 $f' = \text{DNE}$ $f' = 0$
 min $(-2, 0)$ $(2, 0)$
 max $(0, \sqrt[3]{16})$

$(-\infty, -2)$ $f' = \text{Neg}$, Dec
 $(-2, 0)$ $f' = \text{Pos}$, Inc
 $(0, 2)$ $f' = \text{Neg}$, Dec
 $(2, \infty)$ $f' = \text{Pos}$, Inc

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Find all extrema, & Inc/Dec intervals

3. $\frac{x^4 + 1}{x^2}$

$f = x^2 + x^{-2}$
 $f' = 2x - 2x^{-3}$
 $f' = 2x - \frac{2}{x^3} = 0$
 $2x = \frac{2}{x^3}$
 $2x^4 = 2$ $x^4 = 1$ $x = \pm 1$
 $(-\infty, -1)$ $f' = \text{Pos}$ Inc
 $(-1, 0)$ $f' = \text{Neg}$ Dec
 $(0, 1)$ $f' = \text{Pos}$ Inc
 $(1, \infty)$ $f' = \text{Neg}$ Dec
 min $(-1, 2)$ $(1, 2)$
 max $(0, \sqrt[3]{16})$

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