3.7 Optimization Problems

-- Finding the "best" solution to a problem

-- Locate the max or min of an eqn.

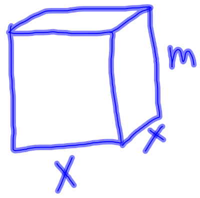
-- where $f'(x) = 0$, or $f'(x)$ DNE

Primary Equation

-- the eqn to be optimized

Secondary Equation

-- any eqn(s) needed for substitution into the primary eqn ex1) an open top box is to be made with a square base and a surface area of 108 in^3. What dimensions will give largest volume?



primary

$V = lwh$

$V = x^2 \cdot m$

secondary

$SA = 108$

$SA = \text{bottom} + 4\,\text{sides}$

$108 = x^2 + 4xm$

$108 - x^2 = 4xm$

$m = \frac{108 - x^2}{4x}$

$V = x^2 m$

$= \cancel{x^2}\left(\frac{108 - x^2}{4\cancel{x}}\right)$

$V = \frac{108x - x^3}{4}$

$V = 27x - \frac{1}{4}x^3$

$V' = 27 - \frac{3}{4}x^2 = 0$

$27 = \frac{3}{4}x^2$

$27\left(\frac{4}{3}\right) = x^2$

$x^2 = 36$

$x = 6$ in

$m = \frac{108 - x^2}{4x}$

$= \frac{108 - 6^2}{24} = 3$ in

Box: $6 \times 6 \times 3$ in

2) Find the points on the graph of $y = 4 - x^2$ that are closest to (0, 2)

(x, y)

min. Dist.

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$D = \sqrt{(x-0)^2 + (y-2)^2}$$

$$= \sqrt{x^2 + (4 - x^2 - 2)^2}$$

$$= \sqrt{x^2 + (2 - x^2)^2}$$

$$= \sqrt{x^2 + 4 - 4x^2 + x^4}$$

$$D = \sqrt{x^4 - 3x^2 + 4}$$

$$D^2 = x^4 - 3x^2 + 4$$

$$(D^2)' = 4x^3 - 6x = 0$$

$$2x(2x^2 - 3) = 0$$

$$x = 0 \quad , \quad x^2 = 3/2$$

$$x = \pm\sqrt{3/2}$$

$$(-\infty, -\sqrt{3/2})\ (-\sqrt{3/2}, 0)\ (0, \sqrt{3/2})\ (\sqrt{3/2}, \infty)$$

$-$	$+$	$-$	$+$
Dec	Inc	Dec	Inc

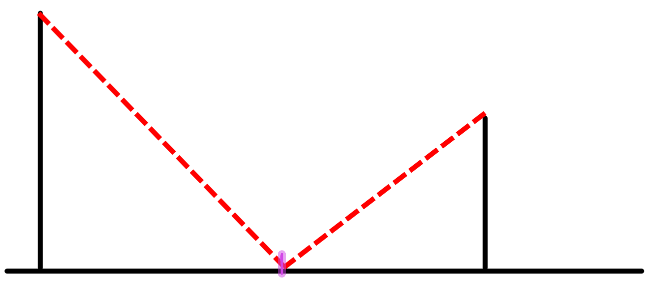
$$x = \pm\sqrt{3/2}$$

$y = 4 - x^2$

$4 - (3/2)$

$$\boxed{\left(\pm\sqrt{3/2}, \ 5/2\right)}$$

3) Two posts are 30 feet apart. One is 12 feet high, & the other is 28 feet high.
They are connected to the ground with wires running from the top of each post to a stake located on the ground between them.
Where should the stake be located to use the smallest amount of wire?



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2, 6, 7, 9, 11, 12

15, 21