3.9 Differentials

Differential of y

Let y = f(x) be a differentiable function and dx is a nonzero Real Number,

Then: dy = f'(x)·dx

$f'(x) = \frac{dy}{dx}$

$f'(x) \cdot dx = dy$

dy = differential of y
Δy = actual change in y = f(x+Δx) - f(x)

Δy ≈ dy

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ex1) $y = x^2$

Find dy when x=1 and dx=.01

Compare this to the exact value of Δy

$dy = f'(x) \cdot dx$
$= 2x \cdot dx$
$= 2(1)(.01)$
$dy = .02$

$\Delta y = f(x+\Delta x) - f(x)$
$= f(1.01) - f(1)$
$= 1.0201 - 1$
$= .0201$

1b) Use the eqn of the tangent line to $f(x) = x^2$ at x=1 to approximate f(1.01)
This is the Linear approximation of f at x=1

$f = x^2$
$f' = 2x$ = slope
$f'(1) = 2(1) = 2$
(1, 1)

Tan:
$y - 1 = 2(x-1)$
$y = 2x - 1$

$f(1.01) \approx 2(1.01) - 1$
$\approx 2.02 - 1$
≈ 1.02

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2) Find dy
for $y = 1 - 2x^2$
when x=0 & dx = -.1

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Error

Propagated Error = dy

The amount a measurement (Δy) may be off due to an error in measurement (Δx)

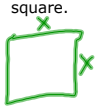
% Error

The ratio of the error to the original measured value:

Relative Error = dy / y
Percent Error = Rel. Error * 100

ex3) A square with sides of length 4 cm may be off by at most ±.02 cm when measured.

Find the propagated (actual) error and the percent error in calculating the Area of the square.



$A = x^2$
$dA = 2x \cdot dx$
$= 2(4)(\pm .02)$
$dA = \pm .16 \text{ cm}^2$

$\frac{dA}{A} = \frac{\pm .16}{16} = \pm .01$
$\pm 1\%$

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ex4) Volume of a sphere:
The radius is 7 ft with a possible error of $\pm .05$ ft

Find the propagated & % error in volume

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5) Use differentials to approximate $\sqrt{25.5}$

$y = \sqrt{x}$

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6) if $f(x) = \sin(0.25\pi x)$, use differentials to approximate $f(3.01)$

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