5.5 Other bases of Exponential Functions

a^x

Rules: 1) $a^0 = 1$

2) $a^x \cdot a^y = a^{x+y}$

3) $a^x / a^y = a^{x-y}$

4) $(a^x)^y = a^{xy}$

Log Properties:

1) $y = a^x$ if & only if $x = \log_a y$

2) $a^{\log_a x} = x$

3) $\log_a a^x = x$

Log Rules

1) $\log_a 1 = 0$

2) $\log_a x \cdot y = \log_a x + \log_a y$

3) $\log_a (x/y) = \log_a x - \log_a y$

4) $\log_a x^n = n \cdot \log_a x$

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Derivatives

1) $\frac{d}{dx} a^x = (\ln a) \cdot a^x$

2) $\frac{d}{dx} a^u = (\ln a) \cdot a^u \cdot du$

3) $\frac{d}{dx} \log_a x = \frac{1}{(\ln a)\, x}$

4) $\frac{d}{dx} \log_a u = \frac{1}{(\ln a)\, u} \cdot du$

ex1) Solve

$3^x = \frac{1}{81} \rightarrow \log_3 \frac{1}{81} = x$

$= \frac{\ln \frac{1}{81}}{\ln 3}$

$\therefore x = -4$

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2) $\log_2 x = -4$

$2^{-4} = x$

$\frac{1}{16} = x$

3) $y = 2^x$

$y' = 2^x \cdot \ln 2$

4) $y = 3^{5x}$

$y' = \ln 3 \cdot 3^{5x} \cdot 5$

5) $y = 4^x - 2^{-3x}$

$y' = \ln 4 \cdot 4^x - \ln 2 \cdot 2^{-3x}(-3)$

$= \ln 4 \cdot 4^x + \frac{3 \ln 2}{2^{3x}}$

6) $\log_7(3x)$

$\frac{1}{\ln 7 \cdot 3x} \cdot 3$

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ex 1) $y = \log_2(5x+3)$

$$y' = \frac{1}{\ln 2 \cdot (5x+3)} \cdot 5 = \frac{5}{\ln 2 (5x+3)}$$

2) $\log_3 \frac{1}{27}$ → solve $3^x = \frac{1}{27}$

$= -3$

3) $\log_8 2$

$8^x = 2$

$x = \frac{1}{3}$

4) $y = 5^{x-2}$

$y' = \ln 5 \cdot 5^{x-2} \cdot 1$

5) $f(x) = \dfrac{7^{2x}}{x} = 7^{2x} \cdot x^{-1}$

$f' =$ first (d second) + second (d first)

$= 7^{2x}(-1x^{-2}) + x^{-1} \cdot (\ln 7 \cdot 7^{2x} \cdot 2)$

$-\dfrac{7^{2x}}{x^2} + \dfrac{2\ln 7 \cdot 7^{2x}}{x}$

6) $y = \log_2 \sqrt[3]{4x-3}$

$y' = ?$ $\quad y = \frac{1}{3}\log_2(4x-3)$

$\dfrac{4}{3\ln 2(4x-3)}$

$\frac{1}{3} \cdot \dfrac{1}{\ln 2(4x-3)} \cdot 4$

7) $y = \log\left(\frac{x\sqrt{3x+1}}{2}\right)$

$= \log x \cdot \sqrt{3x+1} - \log 2$

$y = \log_{10} x + \frac{1}{2}\log_{10}(3x+1) - \log 2$

$y = \frac{1}{\ln 10 \cdot x} + \frac{3}{2\ln 10(3x+1)}$

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