P.3 Functions & Graphs

Relation -- a set of ordered pairs (x, y)

Function -- a relation in which each member of the domain (x) is paired with *exactly one* member of the range (y).
-- a single x cannot be paired w/ more than one y
-- must pass the Vertical Line Test in which all vert. lines can hit the graph no more than once

Implicit form: $x^2 + 2y = 1$
Explicit form: $y = 1/2(1 - x^2)$
Function Notation: $f(x) = 1/2(1 - x^2)$

Domain (x) -- the independent variable
Range (y) -- the dependent variable, also called the "image" of x under *f*

State the domain of the following functions:

1) $f(x) = \frac{1}{x^2 - 4}$

Denom. $\neq 0$
$x \neq \pm 2$
All $\mathbb{R}$ except ± 2
$(-\infty, -2)(-2, 2)(2, \infty)$

2) $g(x) = \sqrt{x - 7}$

$\sqrt{\ } \geq 0$
$x - 7 \geq 0$
$x \geq 7$
All $\mathbb{R} \geq 7$
$[7, \infty)$

3) $h(x) = 2x - 4x^3 + 8$

All $\mathbb{R}$
$(-\infty, \infty)$

4) $j(x) = \frac{x + 3}{\sqrt{x + 2}}$

$\sqrt{x+2} \geq 0$
$x \geq -2$
All $\mathbb{R}$ $x > -2$
$(-2, \infty)$

5) Calculate the following values from the above functions

$f(-1) = \frac{1}{x^2 - 4}$
$= -\frac{1}{3}$

$g(3) = \sqrt{x - 7}$
$= 2i$

$h(a) = 2x - 4x^3 + 8$
$2a - 4a^3 + 8$

$j(x - 2) = \frac{x + 3}{\sqrt{x + 2}}$
$= \frac{x + 1}{\sqrt{x}}$

6) Let $f(x) = x^2 + 7$, calculate the following:

$f(3a) =$ $9a^2 + 7$

$f(b - 1) =$ $b^2 - 2b + 8$

Difference Quotient:

$\frac{f(x + h) - f(x)}{h} = \frac{[(x+h)^2 + 7] - [x^2 + 7]}{h}$

$\frac{x^2 + 2hx + h^2 + 7 - x^2 - 7}{h}$

$2x + h$

Difference Quotient

-- a formula for the slope of any curve at a point.

$$= \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

or:

$$= \frac{f(x + h) - f(x)}{h}, \quad \text{where } h = \Delta x$$

1) $f(x) = 7x - 4$

$$\frac{[7(x+h) - 4] - [7x - 4]}{h}$$

$$\frac{7x + 7h - 4 - 7x + 4}{h} = \frac{7h}{h} = 7$$

2) $g(x) = x^2 + 3x - 5$, find the slope when $x = 2$

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{[(x+h)^2 + 3(x+h) - 5] - [x^2 + 3x - 5]}{h}$$

$$\frac{x^2 + 2xh + h^2 + 3x + 3h - 5 - x^2 - 3x + 5}{h}$$

$$= 2x + h + 3$$

@ $x = 2$:

$h + 7$

Piecewise Functions -- functions defined by more than one equation.

ex: $f(x) = \begin{cases} 4 - x, & \text{if } x < 1 \\ \sqrt{x - 1}, & \text{if } x > 1 \end{cases}$

$f(-2) =$ $= 4 - (-2)$ $= 6$

$f(1) =$ $\emptyset$

$f(5) =$ $\sqrt{5-1}$ $= 2$

HW: p28, # 2, 4, 8-16 even, 26, 28-32, 34