Algebra II\ Trig: Identities Quiz

Prove each identity.

1. $\csc A \tan A = \sec A$

$$\frac{1}{\cancel{\sin A}} \cdot \frac{\cancel{\sin A}}{\cos A} = \frac{1}{\cos A}$$

$$\frac{1}{\cos A} = \frac{1}{\cos A}$$

2. $\cos(x+90^\circ)+\cos(x-90^\circ)=0$

$$\cos x \cos 90^\circ - \sin x \sin 90 + \cos x \cos 90^\circ + \sin x \sin 90^\circ = 0$$

$$\cos x(0) - \sin x(1) + \cos x(0) + \sin x(1) = 0$$

$$0 - \sin x + 0 + \sin x = 0$$

$$0 = 0$$

$$2\cos\frac{x+90^\circ+x-90^\circ}{2}\cos\frac{x+90^\circ-(x-90^\circ)}{2} = 0$$

$$2\cos x \cos 90^\circ = 0$$

$$2\cos x(0) = 0$$

$$0 = 0$$

3. $\cot x - \tan x = \dfrac{\cos 2x}{\sin x \cos x}$

$$\frac{\cos x}{\cos x} \cdot \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

4. $(\cos x - \sin x)(\cos x + \sin x) = \cos 2x$

$$\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x$$

5. $\sin^2\frac{A}{2} = \frac{\csc A - \cot A}{2\csc A}$

$$\left(\pm\sqrt{\frac{1-\cos A}{2}}\right)^2 = \frac{\frac{1}{\sin A} - \frac{\cos A}{\sin A}}{\frac{2}{\sin A}}$$

$$\frac{1-\cos A}{2} = \frac{1-\cos A}{\cancel{\sin A}} \cdot \frac{\cancel{\sin A}}{2}$$

$$\frac{1-\cos A}{2} = \frac{1-\cos A}{2}$$

6. $\cot A = \dfrac{\sin 2A}{1-\cos 2A}$

$$\frac{\cos A}{\sin A} = \frac{2\sin A\cos A}{1-(1-2\sin^2 A)}$$

$$= \frac{2\sin A\cos A}{1-1+2\sin^2 A}$$

$$= \frac{\cancel{2}\cancel{\sin A}\cos A}{\cancel{2}\sin^{\cancel{2}} A}$$

$$\frac{\cos A}{\sin A} = \frac{\cos A}{\sin A}$$

$$\frac{2\sin A\cos A}{1-(\cos^2 A-\sin^2 A)}$$

$$\frac{2\sin A\cos A}{1-\cos^2 A+\sin^2 A}$$

$$\frac{2\sin A\cos A}{\sin^2 A+\sin^2 A}$$

$$\frac{\cancel{2}\cancel{\sin A}\cos A}{\cancel{2}\sin^{\cancel{2}} A}$$

$$\frac{\cos A}{\sin A} = \frac{\cos A}{\sin A}$$

7. $\cot x = \dfrac{\cos 3x + \cos x}{\sin 3x - \sin x}$

$$\frac{\cos x}{\sin x} = \frac{2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}}{2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}}$$

$$= \frac{\cancel{2}\cancel{\cos 2x}\cos x}{\cancel{2}\cancel{\cos 2x}\sin x}$$

$$\frac{\cos x}{\sin x} = \frac{\cos x}{\sin x}$$

8. $\tan A \csc A \cos A = 1$

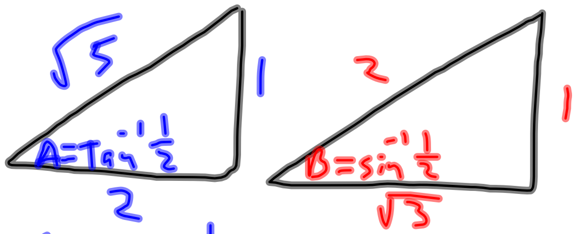
$$\frac{\cancel{\sin A}}{\cancel{\cos A}} \cdot \frac{1}{\cancel{\sin A}} \cdot \frac{\cancel{\cos A}}{1} = 1$$

$$1 = 1$$

Evaluate the expression without using a calculator

9. $\sin\left(\tan^{-1}\frac{1}{2}-\sin^{-1}\frac{1}{2}\right)$

$\sin(A-B)$



$\sin A=\frac{1}{\sqrt5}$ $\qquad \sin B=\frac{1}{2}$

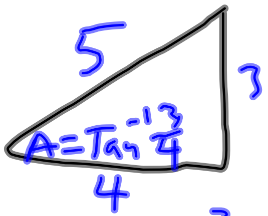
$\cos A=\frac{2}{\sqrt5}$ $\qquad \cos B=\frac{\sqrt3}{2}$

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

$$\frac{1}{\sqrt5}\cdot\frac{\sqrt3}{2}-\frac{2}{\sqrt5}\cdot\frac{1}{2}=\boxed{\frac{\sqrt3-2}{2\sqrt5}}$$

10. $\sin\left(2\tan^{-1}\frac{3}{4}\right)$

$\sin 2A$



$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\sin 2A = 2\sin A\cos A$

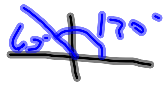
$= \frac{2}{1}\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$

Solve.

11. sin 105˚ sin 15˚

$$\frac{1}{2}\left[\cos(105-15)-\cos(105+15)\right]$$

$$\frac{1}{2}\left(\cos 90^\circ-\cos 120^\circ\right)$$



$$\frac{1}{2}\left(0-\left(-\frac{1}{2}\right)\right)$$

$$\frac{1}{2}\left(\frac{1}{2}\right)$$

$$\boxed{\frac{1}{4}}$$

$$\sin(45+60)\quad\sin(45-30)$$

$$(\sin 45^\circ\cos 60^\circ+\cos 45\sin 60)$$

$$(\sin 45\cos 30^\circ-\cos 45\sin 30)$$

$$\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}+\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$$

$$\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

$$\frac{6-2}{16}=\frac{4}{16}=\frac{1}{4}$$

12. $\sin\frac{7\pi}{12}+\cos\frac{\pi}{12} = \sin\frac{7(180^\circ)}{12} + \cos\frac{180^\circ}{12}$

$\sin 105^\circ + \cos 15^\circ$

$\sin(45^\circ+60^\circ) + \cos(45-30^\circ)$

$\sin 45 \cos 60^\circ + \cos 45^\circ \sin 60^\circ + \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} + \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{2\sqrt{6}+2\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{2}$