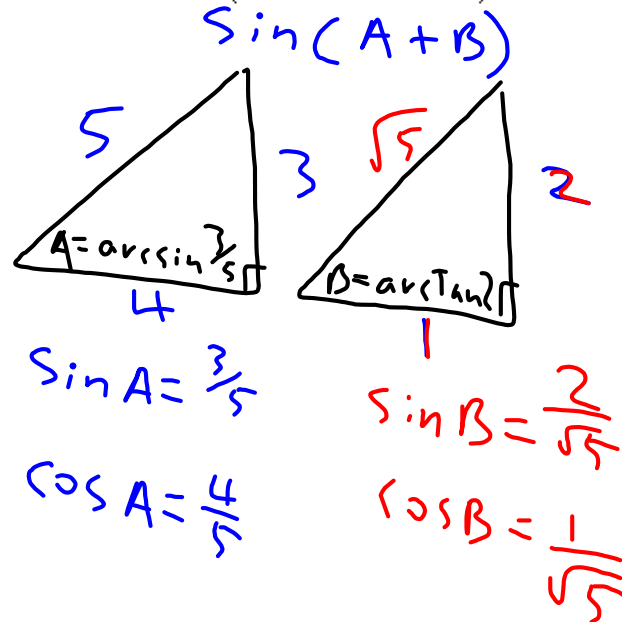


# Additional Trigonometric Identities

**NOTE:**  $\arcsin x = \sin^{-1} x$

Evaluate  $\sin\left(\arcsin\frac{3}{5} + \arctan 2\right)$  without using a calculator.



$\sin(A+B)$

$$\sin A \cos B + \cos A \sin B$$
$$\frac{3}{5} \cdot \frac{1}{\sqrt{5}} + \frac{4}{5} \cdot \frac{2}{\sqrt{5}}$$
$$\frac{3}{5\sqrt{5}} + \frac{8}{5\sqrt{5}} = \frac{11}{5\sqrt{5}}$$

## Product to Sum Formulas

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

Write **10 cos 5x sin 3x** as a sum or difference.

$$10 \cdot \frac{1}{2} [\sin(5x+3x) - \sin(5x-3x)]$$

$$5 [\sin 8x - \sin 2x]$$

## Sum to Product Formulas

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Verify the identity.

$$-\tan x = \frac{\cos 3x - \cos x}{\sin 3x + \sin x}$$

$$-\frac{\sin x}{\cos x} = \frac{-2 \sin \frac{3x+x}{2} \sin \frac{3x-x}{2}}{2 \sin \frac{3x+x}{2} \cos \frac{3x-x}{2}}$$

$$= \frac{-\cancel{2} \sin \cancel{2} x \sin x}{\cancel{2} \sin \cancel{2} x \cos x}$$

$$-\frac{\sin x}{\cos x} = -\frac{\sin x}{\cos x}$$

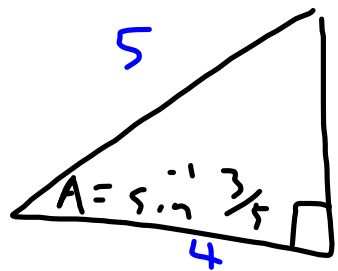
# **Homework**

**Problem Set 5.5 #2-36 even**

**exclude #14 & 24**

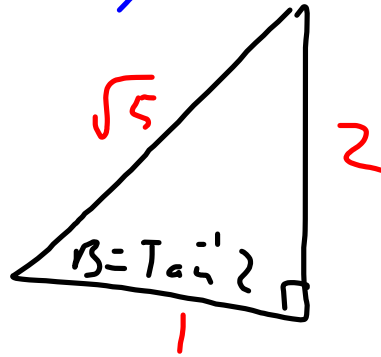
$$2) \cos(\sin^{-1} \frac{3}{5} - \tan^{-1} 2)$$

$$\cos(A - B)$$



$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$



$$\sin B = \frac{2}{\sqrt{5}}$$

$$\cos B = \frac{1}{\sqrt{5}}$$

$$\cos(A - B)$$

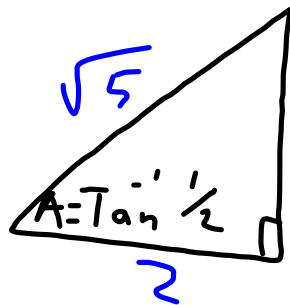
$$\cos A \cos B + \sin A \sin B$$

$$\frac{4}{5} \cdot \frac{1}{\sqrt{5}} + \frac{3}{5} \cdot \frac{2}{\sqrt{5}}$$

$$\frac{4}{5\sqrt{5}} + \frac{6}{5\sqrt{5}} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$$

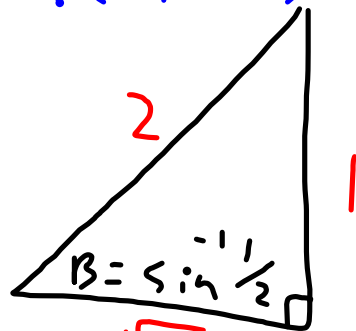
$$4) \sin(\tan^{-1} \frac{1}{2} - \sin^{-1} \frac{1}{2})$$

$$\sin(A-B)$$



$$\sin A = \frac{1}{\sqrt{5}}$$

$$\cos A = \frac{2}{\sqrt{5}}$$



$$\sin B = \frac{1}{2}$$

$$\cos B = \frac{\sqrt{3}}{2}$$

$$\sin(A-B)$$

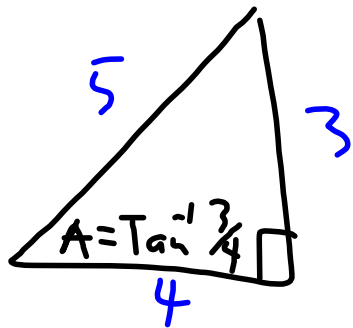
$$\sin A \cos B - \cos A \sin B$$

$$\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{3}}{2} - \frac{2}{\sqrt{5}} \cdot \frac{1}{2}$$

$$\frac{\sqrt{3}}{2\sqrt{5}} - \frac{2}{2\sqrt{5}} = \frac{\sqrt{3} - 2}{2\sqrt{5}}$$

$$6) \sin(2 \tan^{-1} \frac{3}{4})$$

$$\sin 2A$$



$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

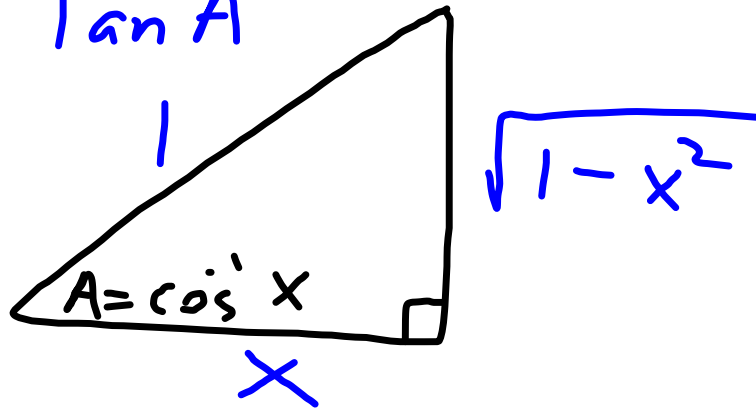
$$\sin 2A$$

$$2 \sin A \cos A$$

$$\frac{2}{1} \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

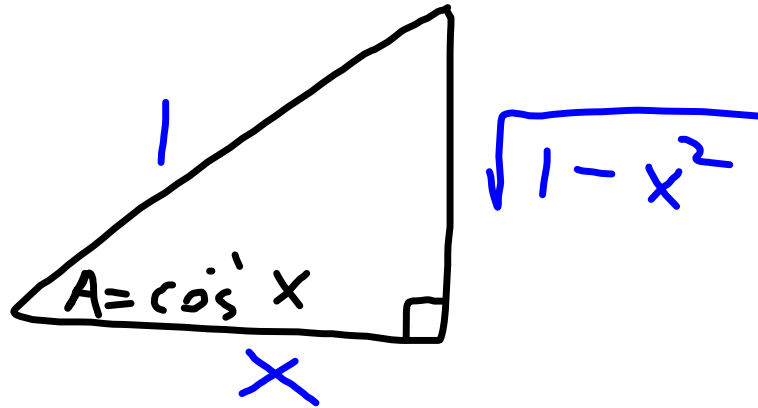
8)  $\tan(\cos^{-1}x)$

$\tan A$



$$\tan A = \frac{\sqrt{1-x^2}}{x}$$

$$10) \sin(2 \cos^{-1} x)$$
$$\sin 2A$$



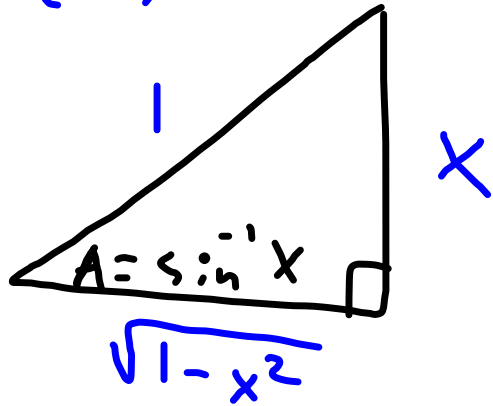
$$\sin A = \sqrt{1-x^2}$$
$$\cos A = x$$

$$\sin 2A = 2 \sin A \cos A = 2 \sqrt{1-x^2} (x)$$

$$\sin 2A = 2x \sqrt{1-x^2}$$

$$12) \cos(2 \sin^{-1} x)$$

$$\cos 2A$$



$$\sin A = x$$

$$\cos A = \sqrt{1-x^2}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$(\sqrt{1-x^2})^2 - (x)^2$$

$$1-x^2-x^2$$

$$\boxed{1-2x^2}$$

$$16) 10 \sin 5x \sin 3x$$

$$10 \cdot \frac{1}{2} [\cos(5x-3x) - \cos(5x+3x)]$$

$$5 [\cos 2x - \cos 8x]$$

$$18) \cos 2x \sin 8x$$

$$\frac{1}{2} [\sin(2x+8x) - \sin(2x-8x)]$$

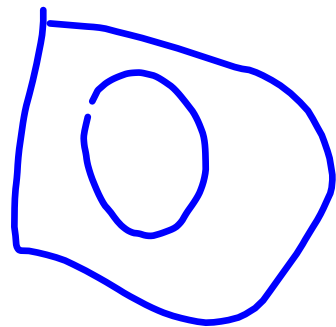
$$\frac{1}{2} [\sin 10x - \sin(-6x)]$$

$$20) \cos 90^\circ \cos 180^\circ$$

$$\frac{1}{2} [\cos(90+180) + \cos(90-180)]$$

$$\downarrow \frac{1}{2} [\cos 270^\circ + \cos(-90^\circ)]$$

$$\frac{1}{2} [0 + 0]$$



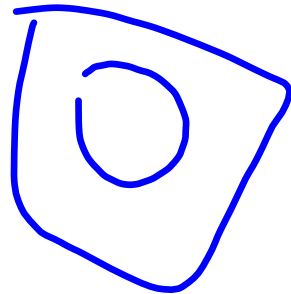
$$22) \cos 3\pi \sin \pi$$

$$\begin{array}{c} \frac{\pi}{2} \\ \pi \text{ --- } \frac{0}{2\pi} \\ \frac{3\pi}{2} \end{array}$$

$$\frac{1}{2} [\sin(3\pi + \pi) - \sin(3\pi - \pi)]$$

$$\frac{1}{2} [\sin 4\pi - \sin 2\pi]$$

$$\frac{1}{2} [0 - 0]$$



$$26) \cos 5x - \cos 3x$$

$$-2 \sin \frac{5x+3x}{2} \sin \frac{5x-3x}{2}$$

$$-2 \sin 4x \sin x$$

$$28) \sin 75^\circ - \sin 15^\circ$$

$$2 \cos \frac{75^\circ + 15^\circ}{2} \sin \frac{75^\circ - 15^\circ}{2}$$

$$2 \cos 45^\circ \sin 30^\circ$$

$$\cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{2}}{2}}$$

$$30) \cos \frac{\pi}{12} + \cos \frac{7\pi}{12}$$

$$\frac{\cancel{\pi}}{2} \cdot \frac{15}{\cancel{\pi}} = 15^\circ$$

$$\cos 15^\circ + \cos 105^\circ$$

$$2 \cos \frac{15^\circ + 105^\circ}{2} \cos \frac{15^\circ - 105^\circ}{2}$$

$$2 \cos 60^\circ \cos (-45^\circ)$$

$$\frac{\cancel{2}}{1} \cdot \frac{1}{\cancel{2}} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{2}}$$

$$32) \cot x = \frac{\cos 3x + \cos x}{\sin 3x - \sin x}$$

$$\frac{\cos x}{\sin x} = \frac{2 \cos \frac{3x+x}{2} \cos \frac{3x-x}{2}}{2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}$$

$$= \frac{\cancel{2} \cos 2x \cos x}{\cancel{2} \cos 2x \sin x}$$

$$\frac{\cos x}{\sin x} = \frac{\cos x}{\sin x}$$

$$34) -\tan 4x = \frac{\cos 3x - \cos 5x}{\sin 3x - \sin 5x}$$

$$\frac{-\sin 4x}{\cos 4x} = \frac{2 \sin \frac{3x+5x}{2} \sin \frac{3x-5x}{2}}{2 \cos \frac{3x+5x}{2} \sin \frac{3x-5x}{2}}$$

$$= \frac{-\cancel{2} \sin 4x \cancel{\sin(-x)}}{\cancel{2} \cos 4x \cancel{\sin(-x)}}$$

$$-\frac{\sin 4x}{\cos 4x} = -\frac{\sin 4x}{\cos 4x}$$

$$36) \cot 2x = \frac{\sin 3x - \sin x}{\cos x - \cos 3x}$$

$$\frac{\cos 2x}{\sin 2x} = \frac{2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}{-2 \sin \frac{x+3x}{2} \sin \frac{x-3x}{2}}$$

$$= \frac{\cancel{2} \cos 2x \sin x}{-\cancel{2} \sin 2x \sin(x)}$$

$$= \frac{\cos 2x \cancel{\sin x}}{+\sin 2x \cancel{\sin x}}$$

$$\frac{\cos 2x}{\sin 2x} = + \frac{\cos 2x}{\sin 2x}$$